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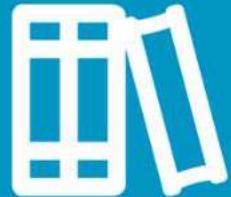
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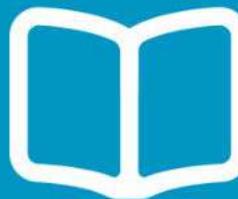
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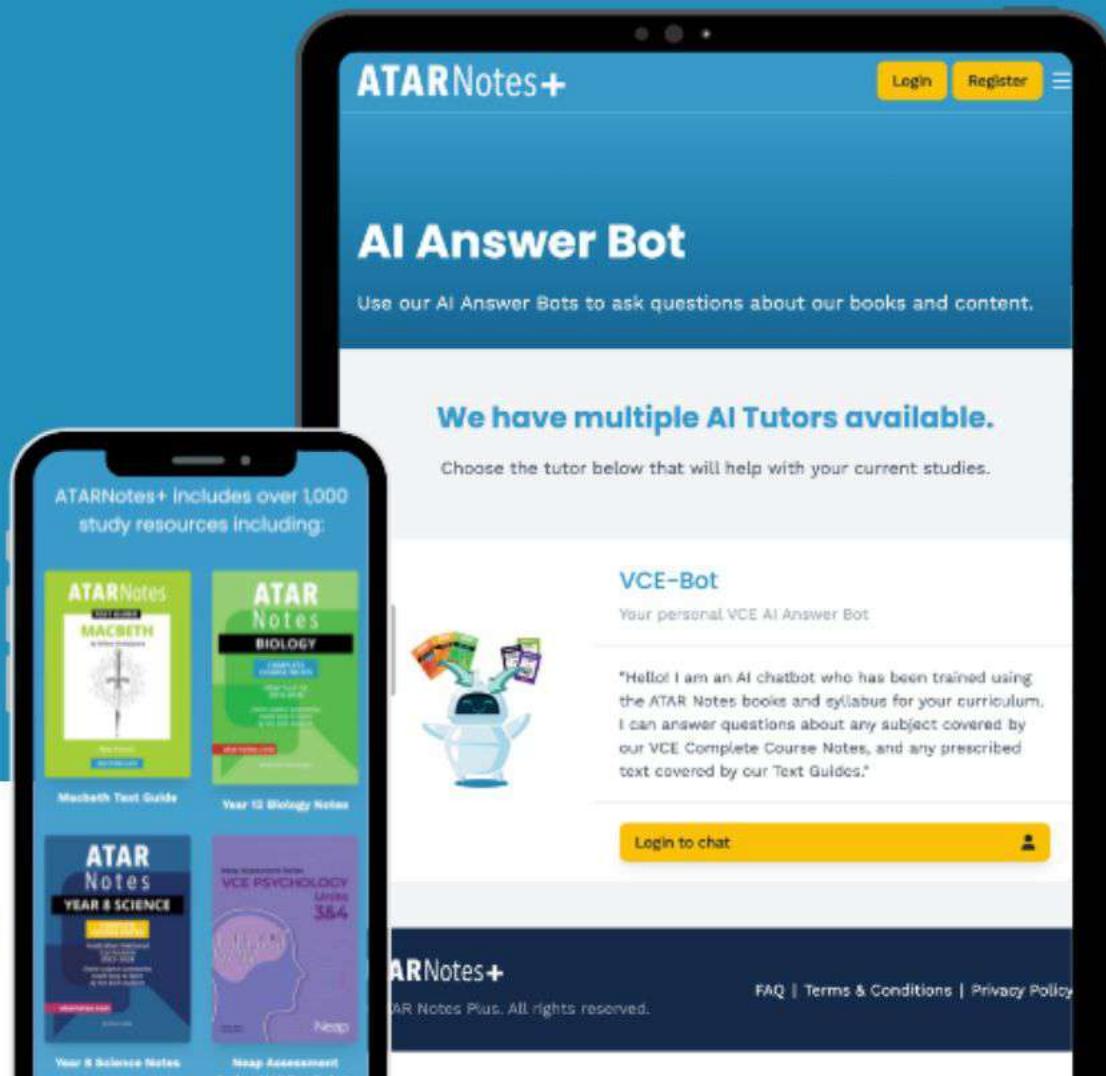
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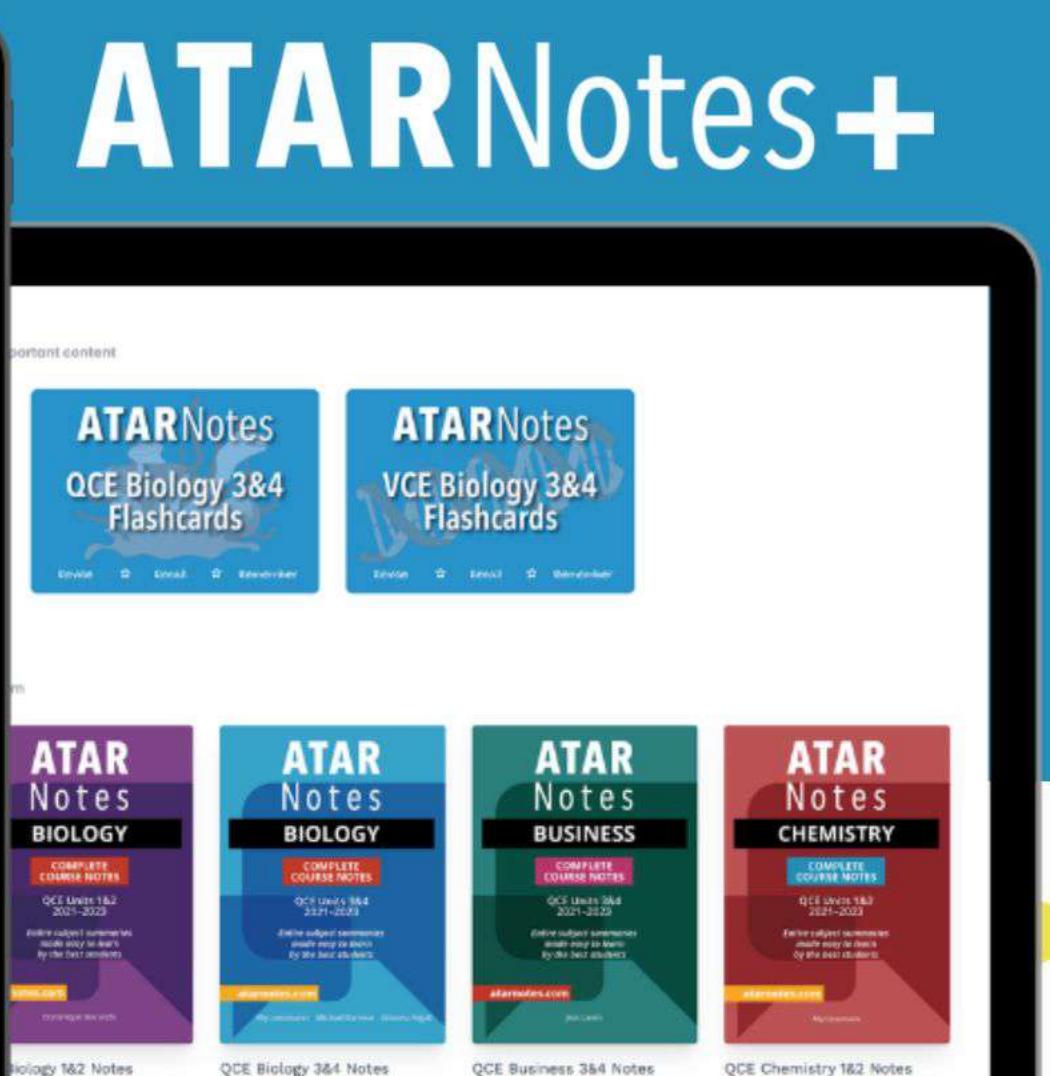
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# ATARNotes

## Year 9 Maths

ATARNotes January Lecture Series

Presented by:  
Michelle W

# Outline

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## Topics to be covered

### Part 1 – Number and Algebra

- Index Laws
- Algebraic Expressions
- Linear Graphs/Equations

### Part 2 – Geometry

- Area, Surface Area and Volume
- Similar Figures
- Pythagoras and Trigonometry

### Part 3 – Chance and Data

- Probability Introduction
- Probability Tools/Diagrams
- Data Representation

### Part 4 – Tips

- Study Tips

## Announcements

- We will be running for an hour
- Please ask any questions via the online chat function

## Goals

---

- Cover some Year 9 math concepts that will be important for this year
- Use the lecture to get an insight into new maths.
- Learn tips, tricks and ways to approach certain problems
- Ask plenty of questions! They will be answered throughout the lecture.

- Indices are a shorthand method of representing a number multiplied by itself many times.
- Index Laws help us simplify indices

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{m \times n}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$
- $a^{1/m} = \sqrt[m]{a}$  or  $a^{n/m} = \sqrt[m]{a^n}$

expanded form

base

exponent/index/power

$$5 \times 5 \times 5 \times 5 = 5^4$$

Consider  $a^m \times a^n$ :

$$a^m \times a^n = \{a \times a \times a \times a \times \dots\} \times \{a \times a \times \dots \times a\}$$

*m factors of a*                            *n factors of a*

$$= a^{m+n}$$

$$\begin{aligned}2^3 \times 2^2 \\&= (2 \times 2 \times 2) \times (2 \times 2) \\&= 2 \times 2 \times 2 \times 2 \times 2 \\&= 2^5\end{aligned}$$

*Simplify the following equation:*  $\frac{8(a^2)^4 \times 5(b^2)^2}{15(a^6)^0 \times 2b^2}$

### STEPS

1. Expand brackets
2. Eliminate multiplication signs
3. Simplify the fraction

$$\begin{aligned} &= \frac{8(a^8) \times 5b^4}{15 \times 2b^2} \\ &= \frac{40a^8b^4}{30b^2} \\ &= \frac{4a^8b^2}{3} \end{aligned}$$

- Scientific is used to express reallyyyyy small and reallyyyyy large numbers in more concise ways – such as 0.0000000067 or 4500000000.
- We write these numbers in the form:

$$a \times 10^m$$

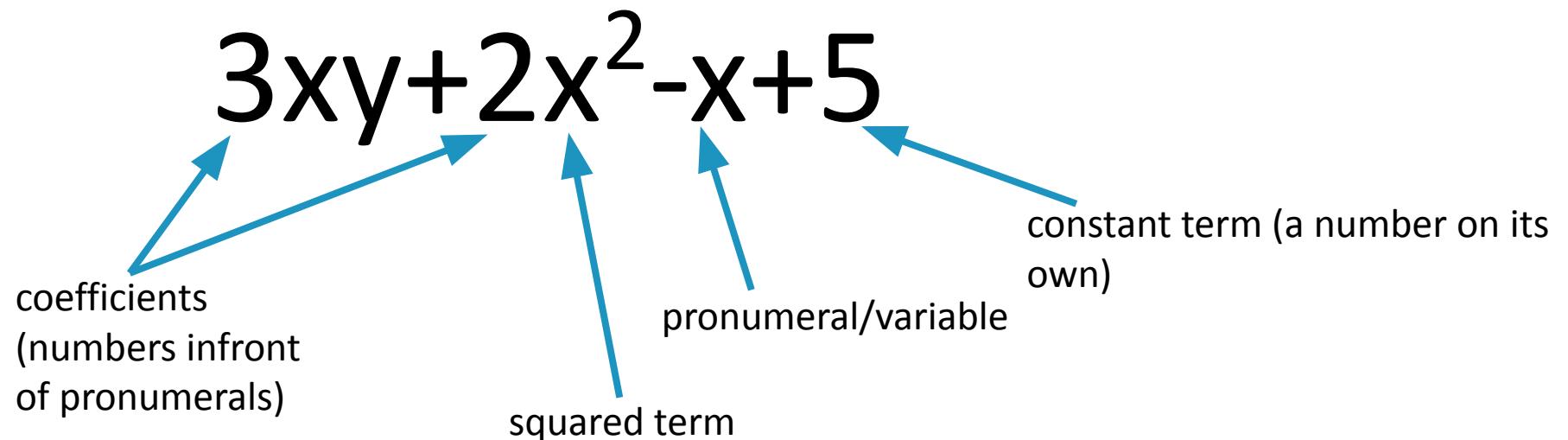
*a is a single digit number but can have decimal places.*

*$m > 0$  for significantly big numbers  
 $m < 0$  for significantly small numbers*

**Express 6400000 in scientific notation.**

**Now you try! Express 0.000034 in scientific notation.**

- Algebra is an extremely important concept in mathematics
- It involves the representation and solution of an unknown variable.
- An **algebraic expression** contains some distinct components.



This is a **four termed expression**.

- We need to get used to forming algebraic expressions when given a written description and vice versa.

**A taxi driver takes an upfront cost of \$5 and charge \$3 per kilometer travelled. Find the cost, \$C of a 2-kilometer taxi drive.**

*Let  $C$  = total cost of taxi drive, and  $k$  = number of kilometers travelled.*

*Therefore,  $C = 5 + 3k$ .*

$$C = 5 + 3(2)$$

$$C = \$11$$

- When we work with algebraic expressions, it is important that we can simplify them.
- We do this by:
  - **Collecting like terms**
  - **Removing multiplication/division signs**
  - **Cancelling out common factors in fractions**

Like terms are terms that are **multiplied** by the **same pronumeral**.

**For example,  $2ab$  and  $5ab$ .**

**$3a$  and  $4a$**

**$3x^2$  and  $4x^2$**

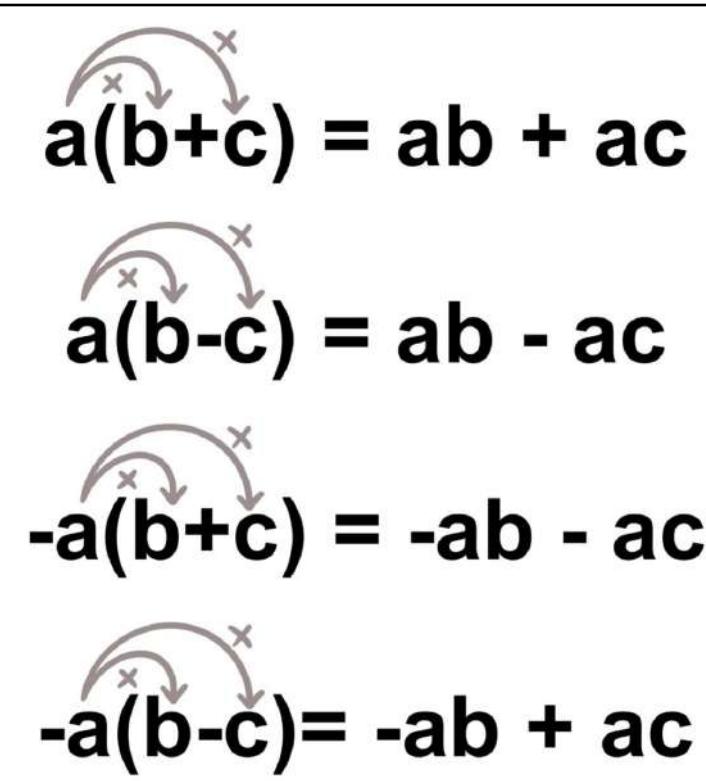
Simplify the expression  $2xy + 5x + 7a^2 + 8xy + 8a^2 + 5$

$$\begin{aligned} & 2xy + 5x + 7a^2 + 8xy + 8a^2 + 5 \\ & 2xy + 8xy + 8a^2 + 7a^2 + 5x + 5 \\ & 10xy + 15a^2 + 5x + 5 \end{aligned}$$

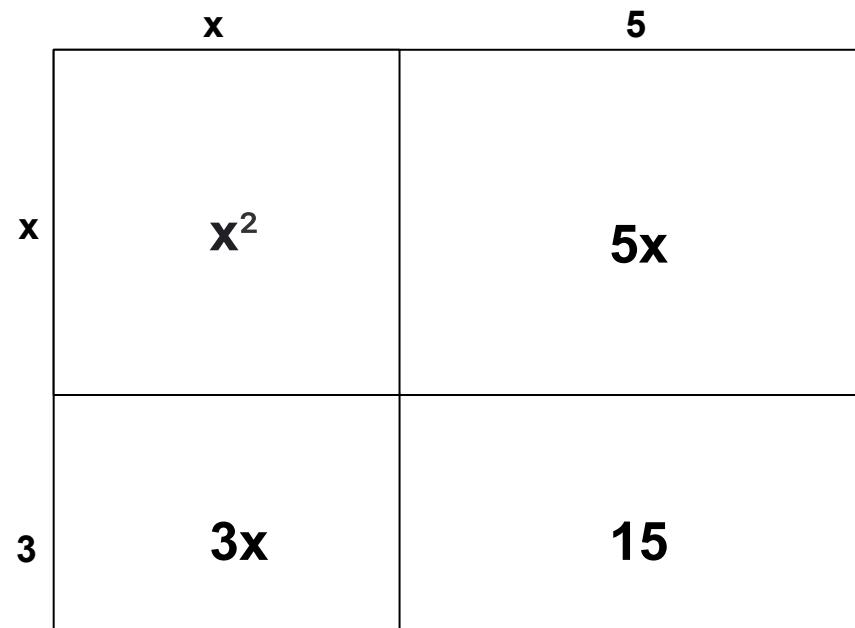
Simplify the expression  $5x \times 8xy + 7b^2 \div 49b$

$$\begin{aligned} & 5x8xy + \frac{7b^2}{49b} \\ & 40x^2 + \frac{7b^2}{49b} \\ & 40x^2 + \frac{b}{7} \end{aligned}$$

- Expanding is the opposite of simplifying – we are essentially removing brackets to create a longer expression.
- Expanding helps us when we need to solving algebraic equations for a specific variable.
- To expand, we use the **distributive law**.


$$a(b+c) = ab + ac$$
$$a(b-c) = ab - ac$$
$$-a(b+c) = -ab - ac$$
$$-a(b-c) = -ab + ac$$

- We can extend the principles of the distributive law to expand out two brackets.



$$\text{AREA} = x^2 + 5x + 3x + 15$$

$$\text{AREA} = x^2 + 8x + 15$$

In other words: Area of rectangle =  $lw$

$$\text{Length} = (x+5)$$

$$\text{Width} = (x+3)$$

$$\text{AREA} = (x+5)(x+3)$$

***So how can we expand this out methodically?***

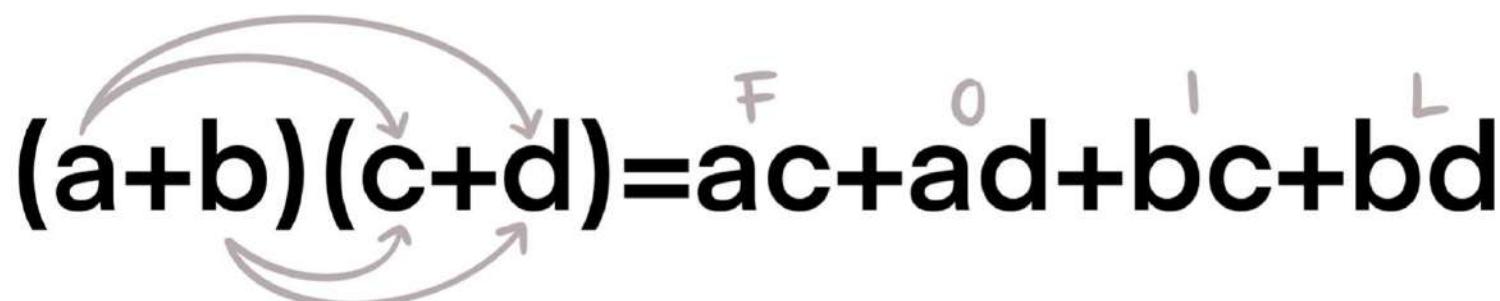
Use this acronym!

**F** - First

**O** - Outer

**I** - Inner

**L** - Last


$$(a+b)(c+d) = ac + ad + bc + bd$$

**Expand**  $(-2x + 1)(x - 5)$ .

*When doing this question, we need to make sure we factor in the negative signs.*

$$\begin{aligned}(-2x + 1)(x - 5) &= (-2x \times x) + (-2x \times -5) + (1 \times x) + (1 \times -5) \\&= -2x^2 + 10x + x - 5 \\&= -2x^2 + 11x - 5\end{aligned}$$

The resulting term is a trinomial expression (it contains 3 terms!)

- A perfect square is a number that is a square of another number: eg. 4, 9, 16 and 25.
- We can even have pronumerals as perfect squares, such as  $x^2$  and  $(a+b)^2$

- We expand perfect squares using the distributive law.

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$$

- We will use this concept when we do factorising.

$$(a + b)(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$$

- Factorising is the process of making an expression more concise. We do this by **writing an expression as the product of its factors**. It is the opposite process of factorising.

$$25x^2 + 5x = 5x(5x+1)$$

expanded form

factorized form

In Year 9, the methods you need to know are: taking out a common factor, factorizing by grouping and factorizing using the difference of perfect squares.

## 1. Factorise by taking out a common factor

$$3x+6x^2+9 \quad (\text{Find the HCF of all terms})$$
$$3(x+2x^2+3)$$

$$ab+3a+a^2$$
$$a(b+3+a)$$

## 2. Factorise by grouping

$$x^2+3x-2x-6 \quad (\text{Group terms that share a common factor})$$
$$x(x+3)-2(x+3) \quad (\text{Take out the common factor})$$
$$(x-2)(x+3)$$

## 2. Factorise by using a difference of perfect squares

$$(a + b)(a - b) = a^2 + b^2$$

*Aim: rewrite expression like the RHS, find a and b values and write as LHS (as that's the factorized form)*

$$x^2-16$$

$$x^2-4^2$$

$$(x+4)(x-4)$$

$$4y^2-16$$

$$4(y^2-16)$$

$$4(y^2-4^2)$$

$$4(y+4)(y-4)$$

- Equations are expressions connected with an equals sign.
- In algebra, we are interested in solving an equation to find the value of a pronumeral.
- The aim is to **isolate** the pronumeral.

### OPPOSITE OPERATIONS – ‘undoing an operation.’

Multiplication -> Division

Addition -> Subtraction

Division -> Multiplication

Subtraction -> Addition

*What operations do we perform to isolate a in the following expressions?*

$$a + 3 = 6$$

$$2a = 8$$

$$a - 5 = 10$$

$$a/4 = 12$$

1. Look at the numbers furthest away from the pronumeral
2. Keep removing those numbers to the other side gradually by undoing the operation.

***Solve the equation  $6m - 8 = 2m$***

$$6m - 2m = 8$$

$$4m = 8$$

$$m = 2$$

***Solve the equation  $\frac{4m}{9} + 4 = 8$***

$$\frac{4m}{9} = 4$$

$$4m = 36$$

$$m = 9$$

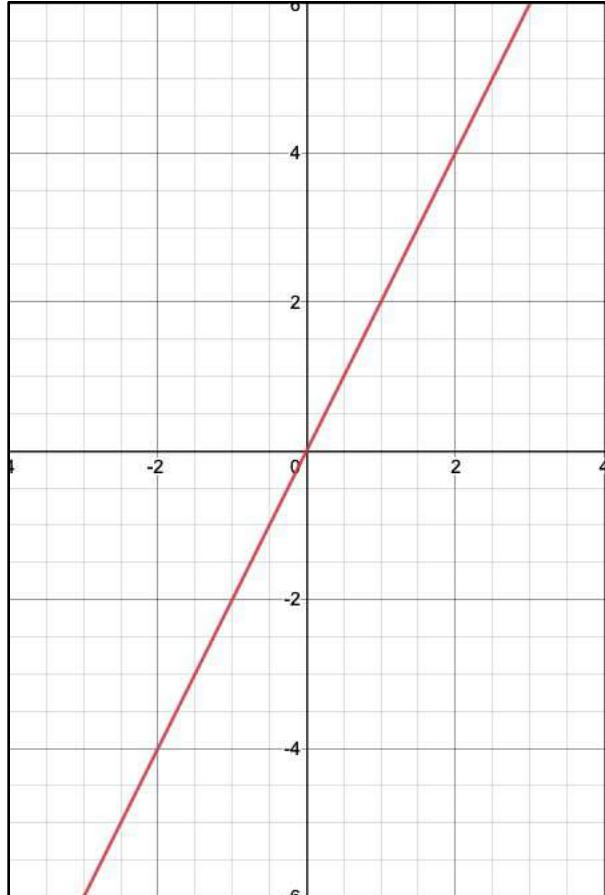
Now you try! See if you can solve this equation.

***Solve  $5(x - 2) = 2x - 13$ , for  $x$ .***

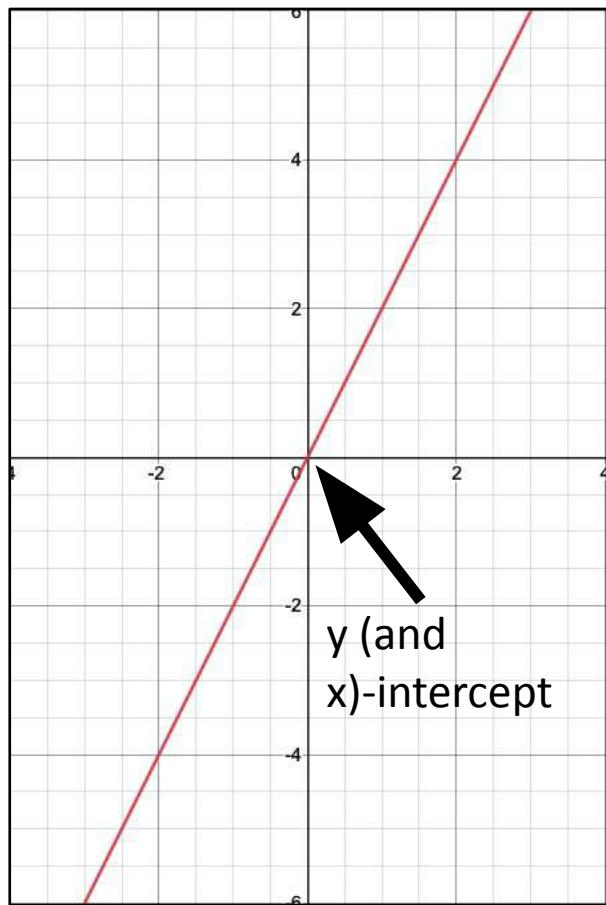
Now you try! See if you can solve this equation.

***Solve  $5(x - 2) = 2x - 13$ , for  $x$ .***

$$\begin{aligned}5x - 10 &= 2x - 13 \\5x - 2x &= 2x - 13 + 10 - 2x \\3x &= -3 \\x &= -1\end{aligned}$$



- This is known as a linear graph. What can you tell me about it? What are some of the distinct features of a linear graph?



$$y=mx+c$$

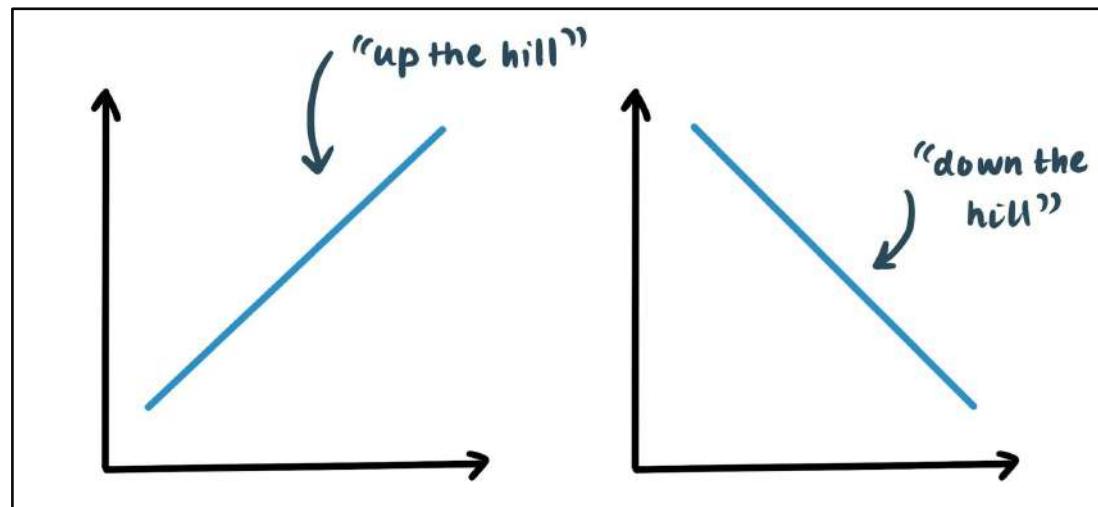
gradient of line

y-intercept

- The gradient of a graph refers to **how steep** the line is.

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

positive gradient



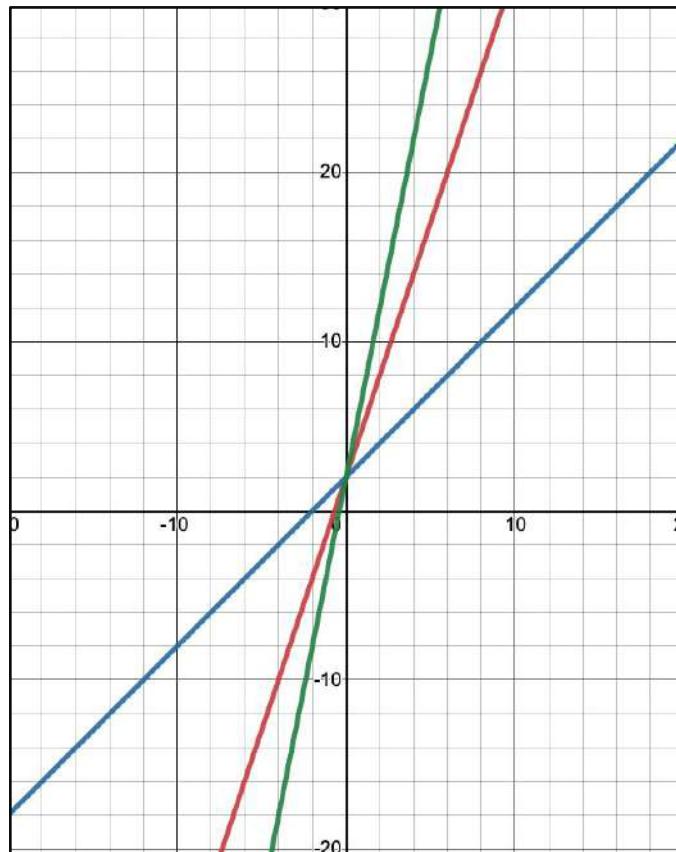
negative gradient

- The larger the gradient of a line, the steeper the graph is

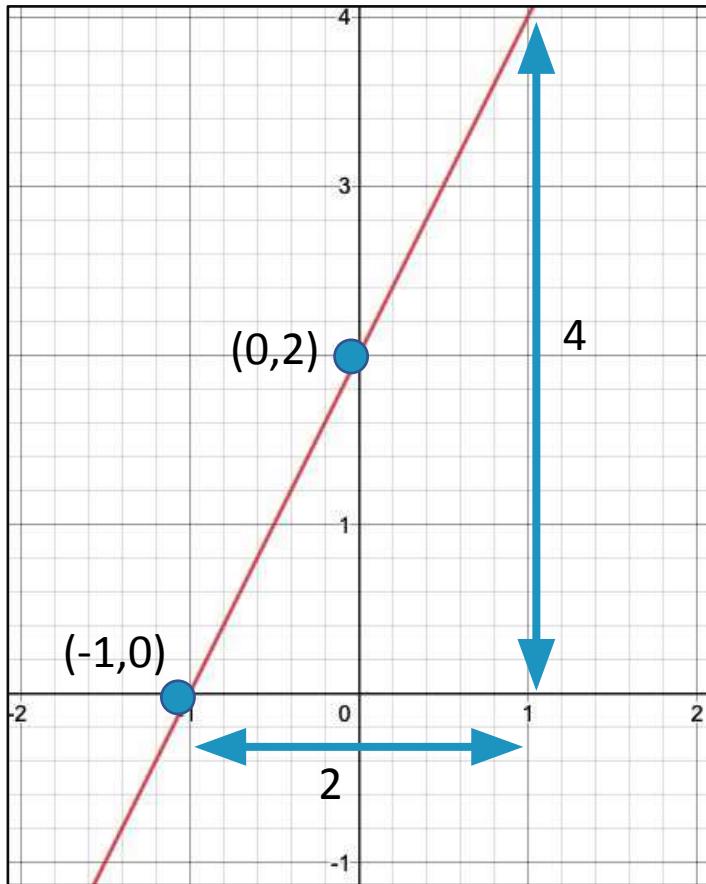
$$y=x+2$$

$$y=3x+2$$

$$y=5x+2$$



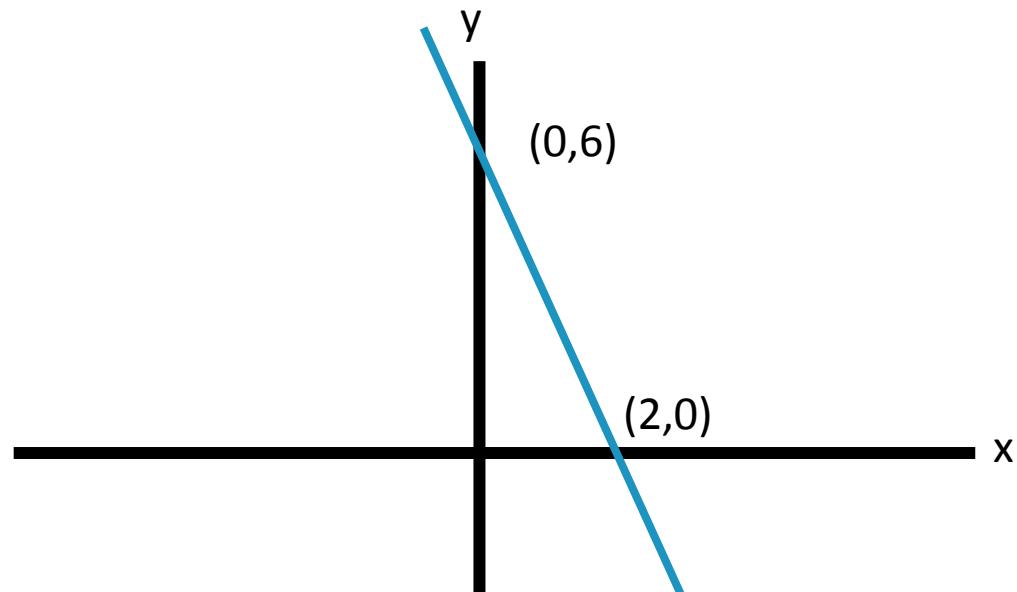
Let's find the gradient of this line.



**Method 1:** 
$$\frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 0}{0 - -1}$$
$$= 2$$

**Method 2:** 
$$\frac{\text{rise}}{\text{run}}$$
$$= \frac{4}{2}$$
$$= 2$$

1. Find x-intercept
2. Find y-intercept
3. Mark them on cartesian plane
4. Join the coordinates



Let's sketch the graph of  $y = -3x + 6$ !

*x-intercept:*

*Let  $y=0$  and solve for  $x$ .*

$$0 = -3x + 6$$

$$-6 = -3x$$

$$x = 2 \quad \square (2,0)$$

*y-intercept:*

*Let  $x=0$  and solve for  $y$ .*

$$y = -3(0) + 6$$

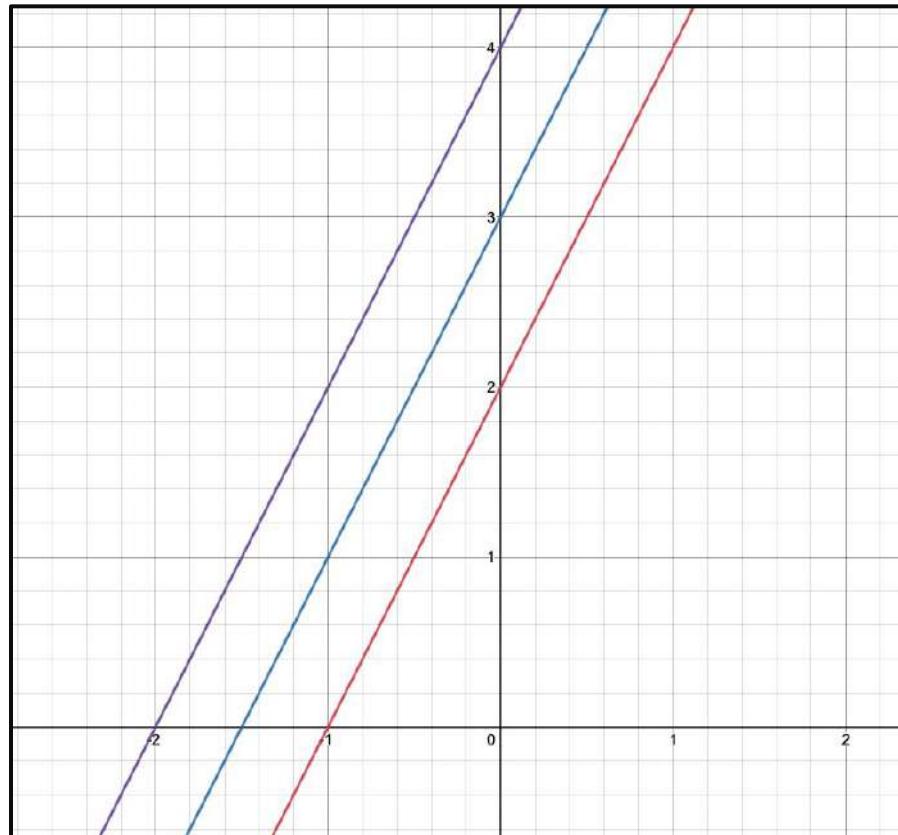
$$y = 6 \quad \square (0,6)$$

- They have the same gradient.

$$y=2x+2$$

$$y=2x+3$$

$$y=2x+4$$



- Helps us calculate the distance between 2 points,  $(x_1, y_1)$  and  $(x_2, y_2)$  on the cartesian plane
- Equation comes from Pythagoras's Theorem

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Helps us calculate the middle coordinate of two points.

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Jack's house is situated at (3,1). His school is situated at (7,5). What is the shortest distance he needs to walk to get to school from his home?

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D = \sqrt{(7 - 3)^2 + (5 - 1)^2}$$

$$D = \sqrt{(4)^2 + (4)^2}$$

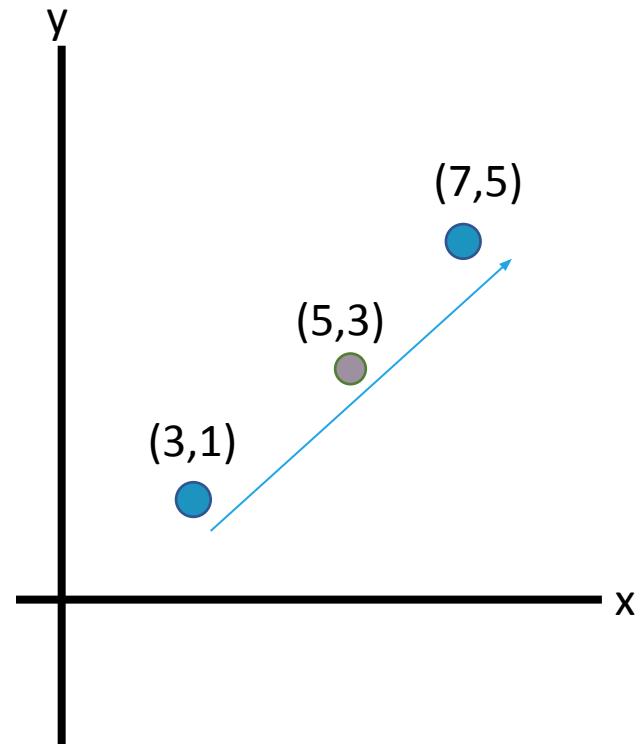
$$D = \sqrt{32} = 4\sqrt{2} \text{ units}$$

Jack would like to take a break when he reaches the halfway point of his journey. What coordinate does this lie at?

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

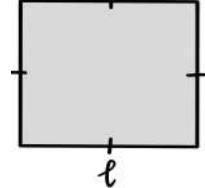
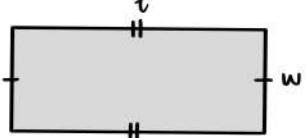
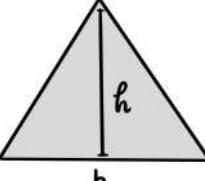
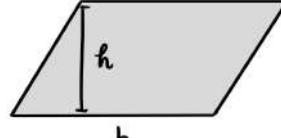
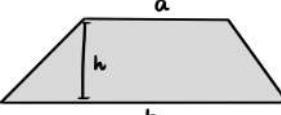
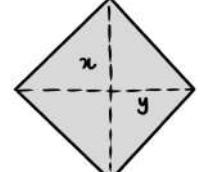
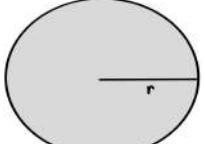
$$M = \left( \frac{7 + 3}{2}, \frac{5 + 1}{2} \right)$$

$$M = (5, 3)$$



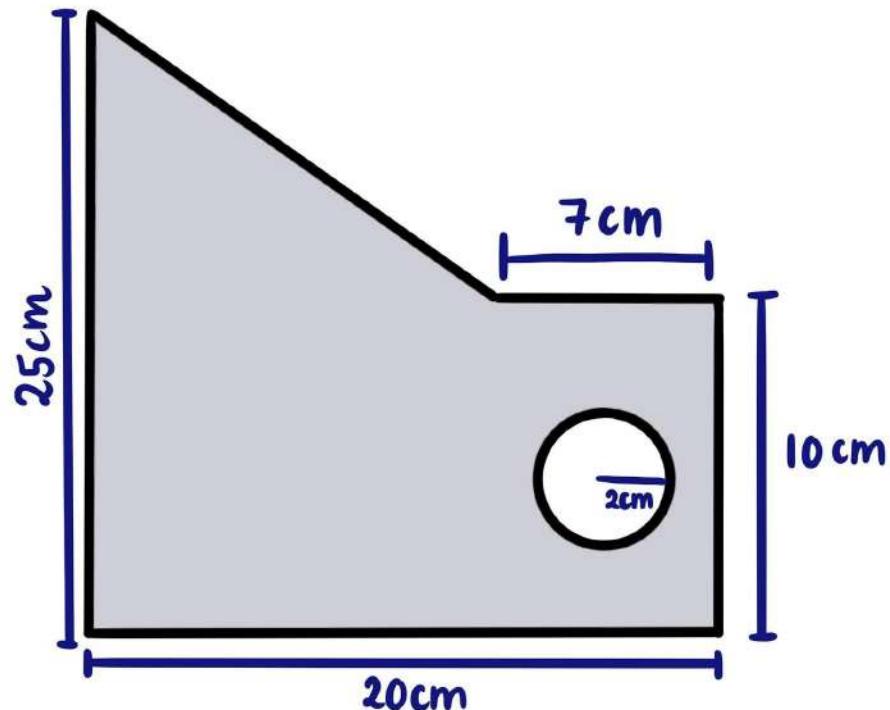
- Practice questions that need many index laws to simplify a single expression.
- Always remember to label any intercepts and axis on your linear graphs!
- Gradually increase the difficulty of the algebra questions when you practice
- Practice graphing a range of different linear graphs and investigate what happens when you change things like vertical translations.

- The area of a 2D shape is the amount of space it occupies.
- We measure area using squared units ( $\text{units}^2$ )
- Recall the area formulas for basic 2D shapes:

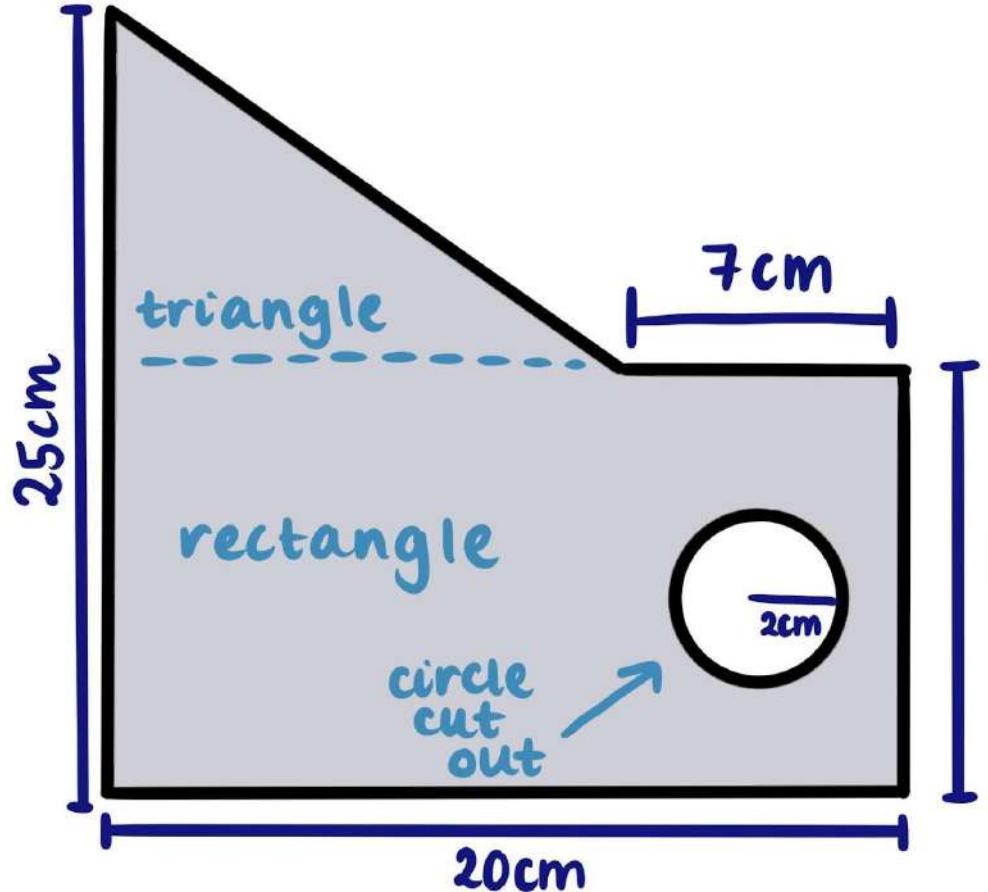
Square	Rectangle	Triangle	Parallelogram
			
Trapezium	Rhombus	Circle	Sector
			

- We can use the basic area formulas to find the area of a composite shape!

Let's find the shaded area of this composite shape:



What shapes can you identify?



**Area of Rectangle:**

$$A = l \times w$$

$$A = 20 \text{ cm} \times 10 \text{ cm}$$

$$A = 200 \text{ cm}^2$$

**Area of Triangle:**

$$A = \frac{1}{2} b \times h$$

$$A = \frac{1}{2} \times (20-7) \times (25-10)$$

$$A = \frac{1}{2} \times 13 \times 15$$

$$A = 97.5 \text{ cm}^2$$

**Area of Circle:**

$$A = \pi r^2$$

$$A = \pi (2)^2$$

$$A = 12.56 \text{ cm}^2$$

**Total Area = area of triangle + area of rectangle – area of circle**

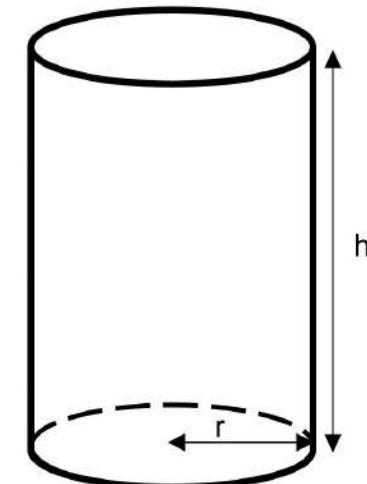
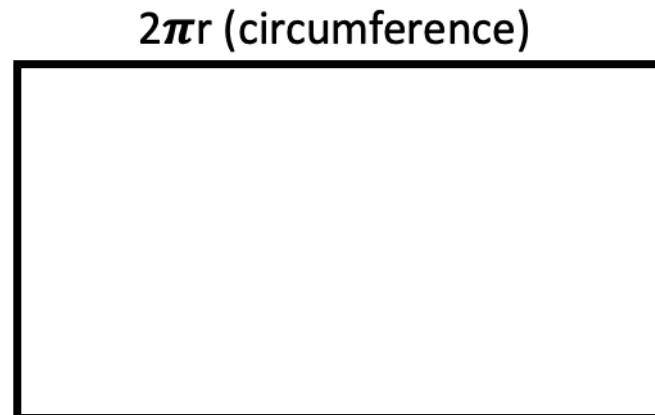
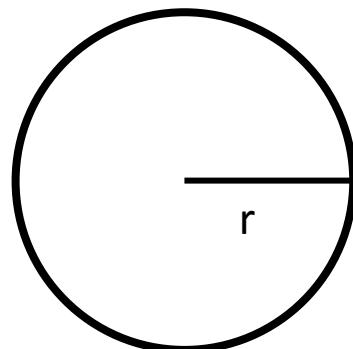
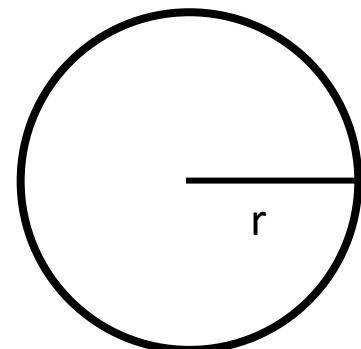
$$= 200 + 97.5 - 12.56$$

$$= 285 \text{ cm}^2$$

- Refers to the sum of the areas of all the individual faces on a **3D shape**.

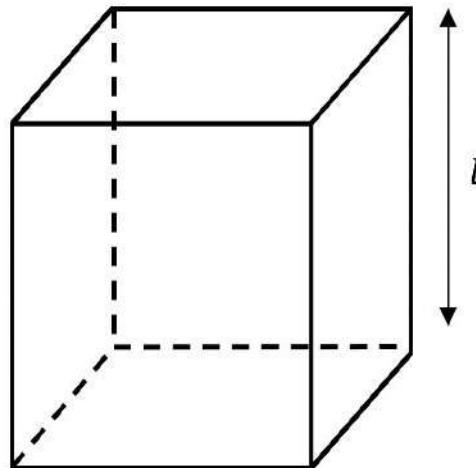
For example, the surface area of this cylinder is given by:

$$SA = 2\pi r^2 + 2\pi r h \text{ units}^2$$



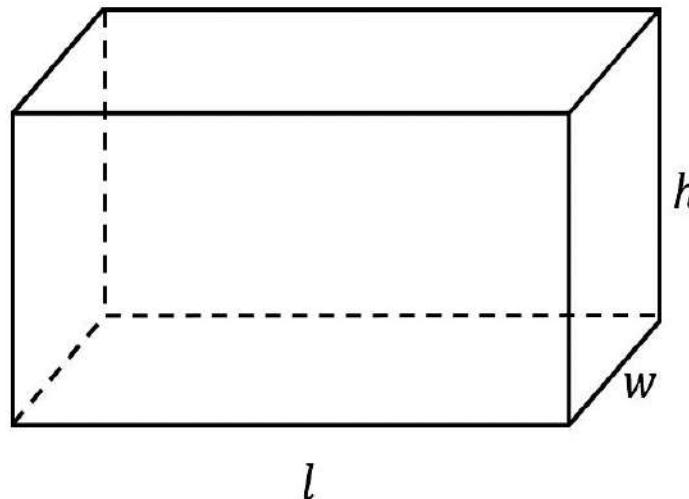
*Surface area of a cube:*

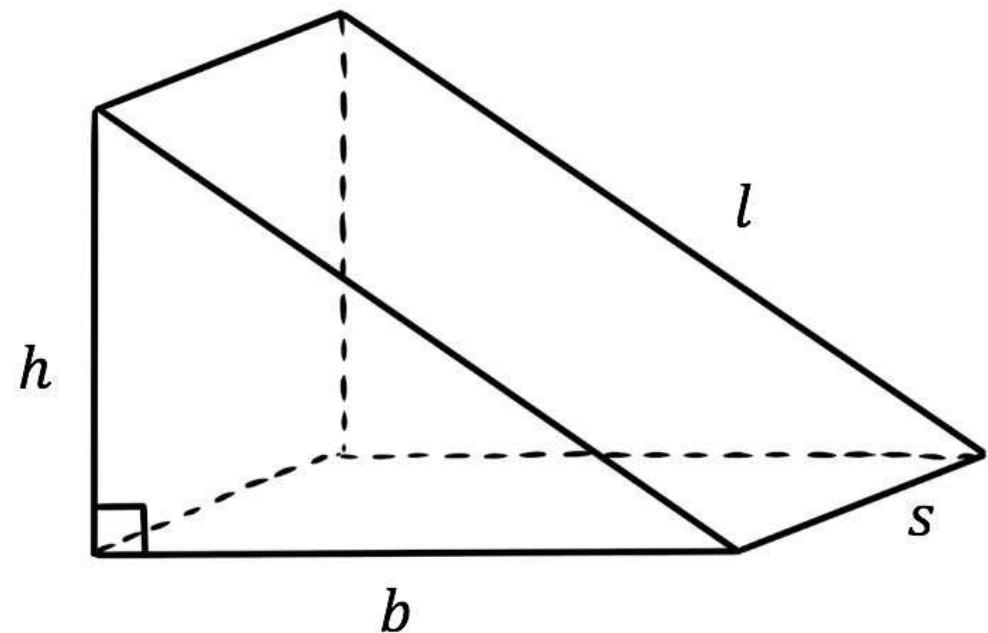
$$SA = 6l^2$$



**Surface area of a rectangular prism:**

$$SA = 2wh + 2lh + 2lw \text{ units}^2$$





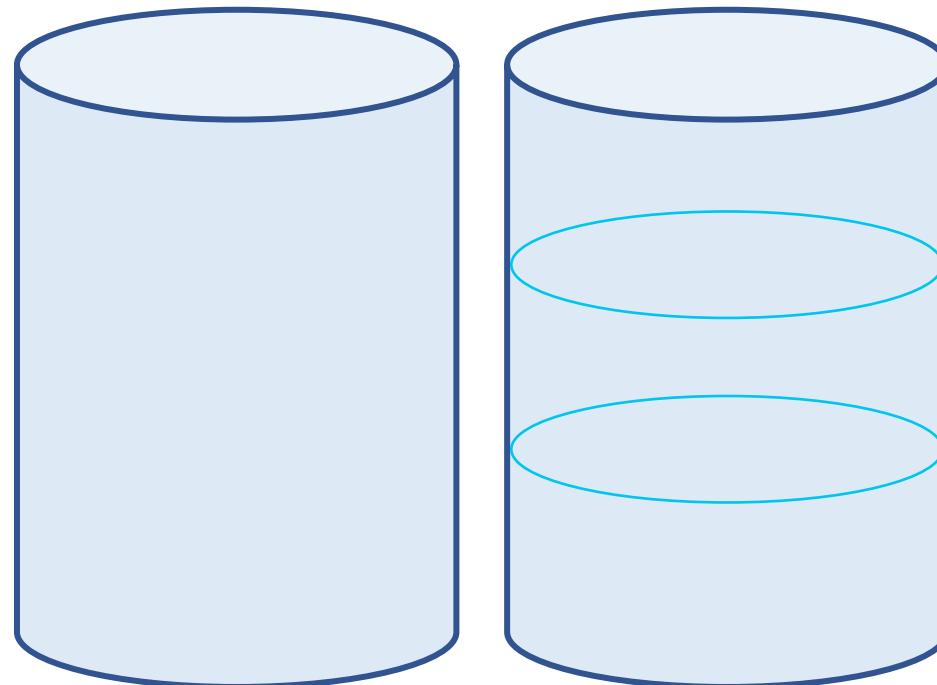
**Surface area of a triangular prism:**

$$SA = bh + bs + hs + ls \text{ units}^2$$

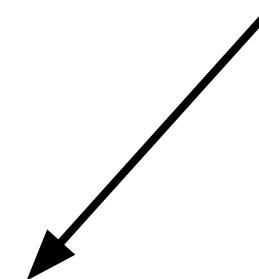
- Area of three rectangles plus two triangles.

- Volume refers to how much content a 3D solid object can hold, or how much space the solid occupies.
- Volume is measured in cubed units ( $\text{m}^3$ ,  $\text{cm}^3$  etc)

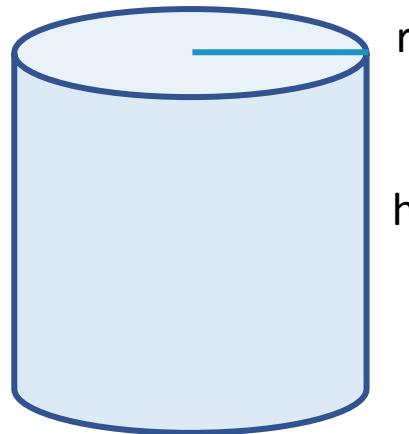
*- Any object with a uniform cross-sectional area has its volume found by multiplying the cross-sectional area by the height of the object.*



cross sectional area (a circle for cylinders!)

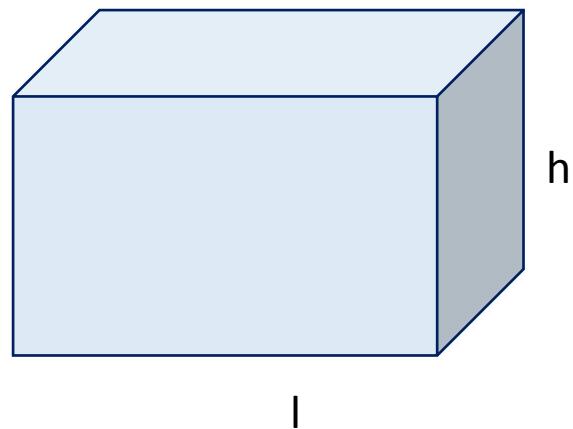


### CYLINDER



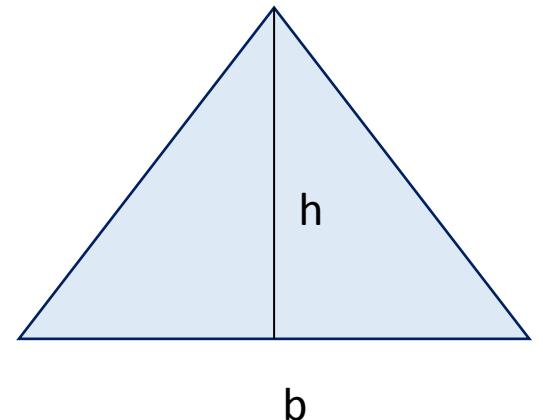
$$V = \pi r^2 h \text{ units}^3$$

### CUBE/RECTANGULAR PRISM



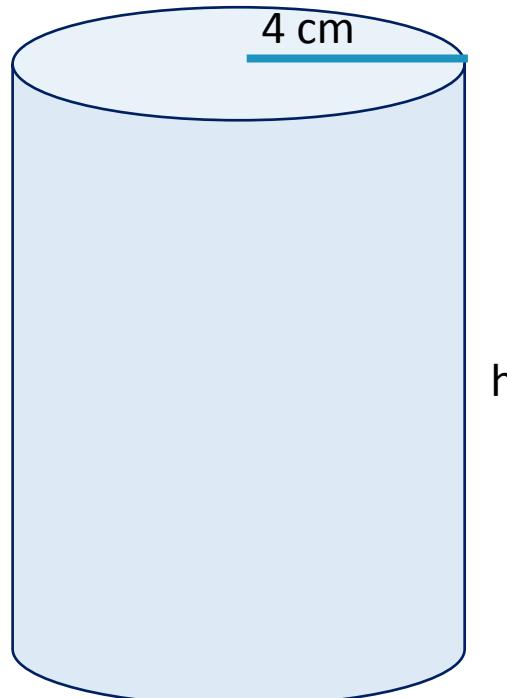
$$V = lwh \text{ units}^3 \text{ or}$$
$$V = l^3$$

### TRIANGULAR PRISM



$$V = \frac{1}{2} bhl \text{ units}^3$$

- Find the height of a cylinder with a radius of 4cm and an area of 200cm<sup>3</sup> to 2 decimal places.



$$V = \pi r^2 h$$

$$200 = \pi 4^2 h$$

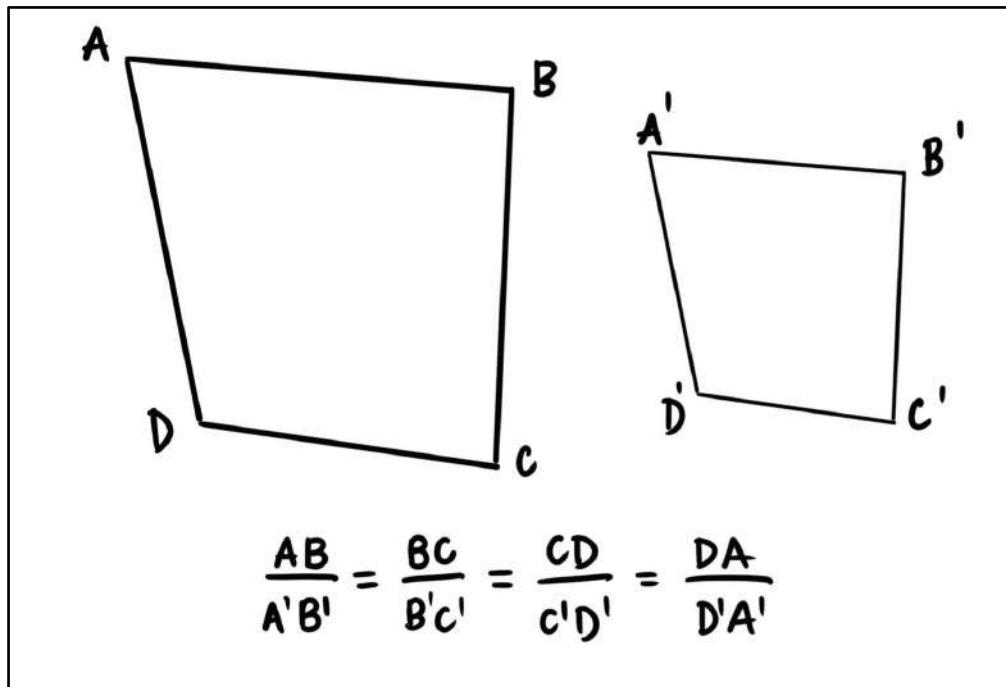
$$200 = 16\pi h$$

$$16\pi h = 200$$

$$\frac{16\pi h}{16\pi} = \frac{200}{16\pi}$$
$$h = \frac{200}{16\pi}$$

$$h = 3.97 \text{ cm}$$

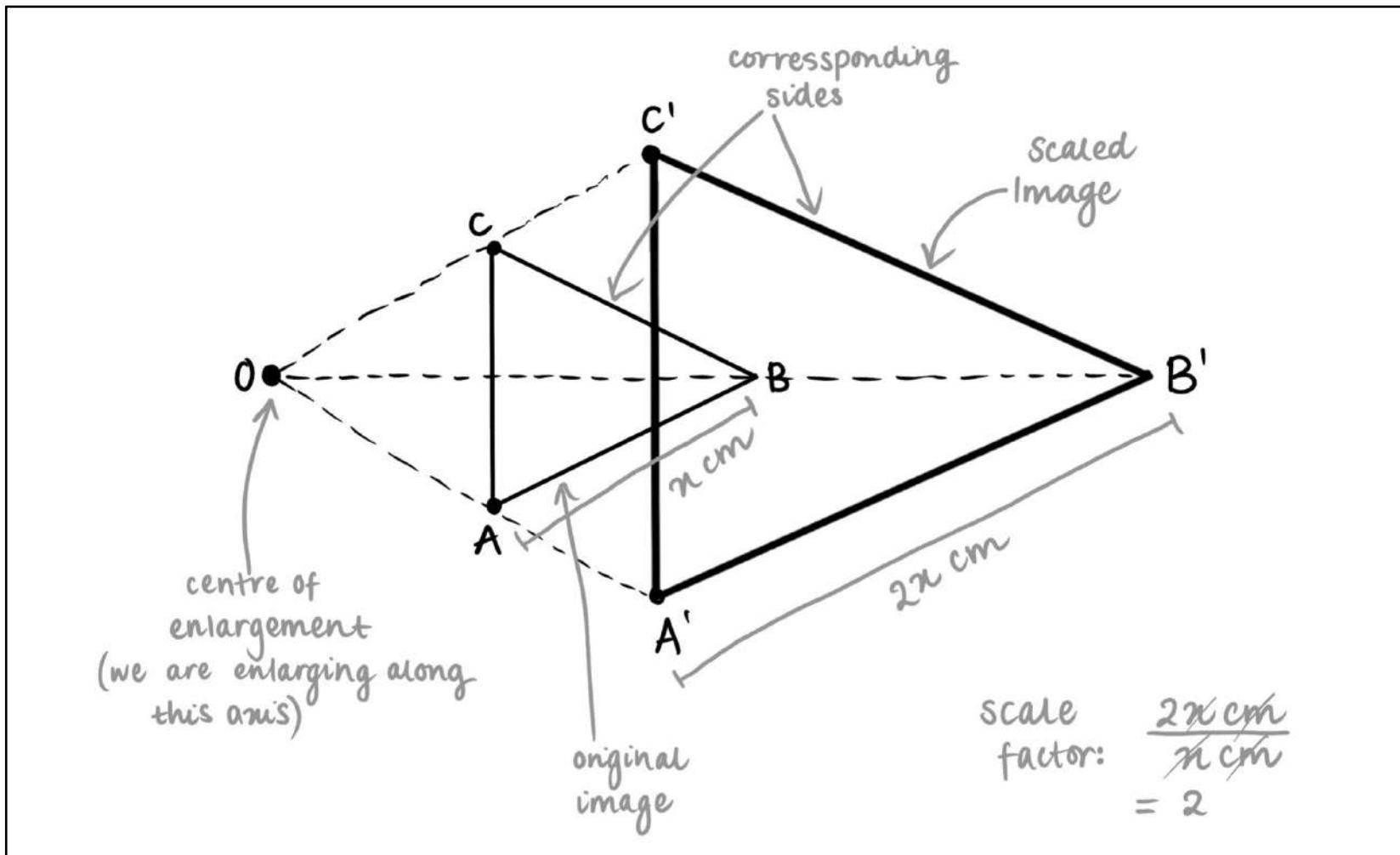
- Enlargement involves a decrease or increase in the size of an object
- The **shape** of the object **doesn't change**, but the **dimensions** do.
- Corresponding angles in each shape are equal.



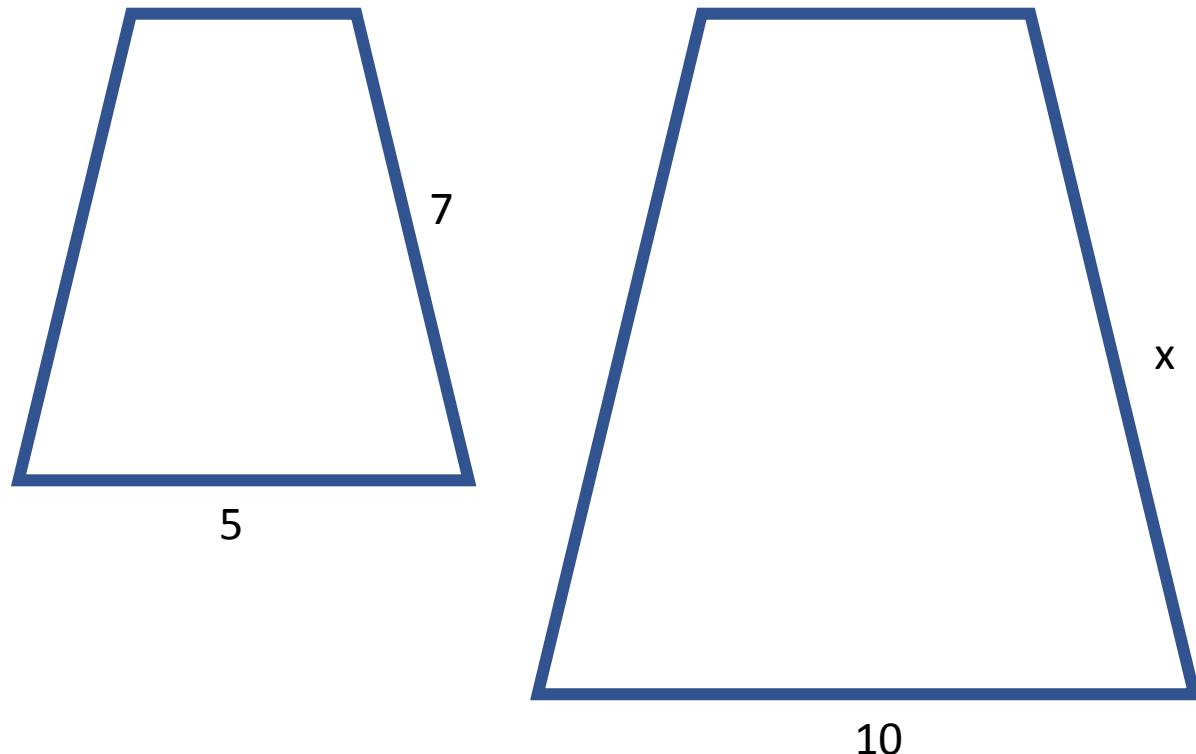
- The scale factor is the ratio of the sides – the size factor we have changed the shape by.

***Scale factor =***  
***length of corresponding side of transformed shape***  
***length of corresponding side of original shape***

Whole number = shape scaled up  
Fraction – shape scaled down  
Equal to 1 = Shapes are congruent



- Find the scale factor and values of the pronumerals for the following similar figures.



$$\text{Scale factor} = \frac{10}{5} = 2$$

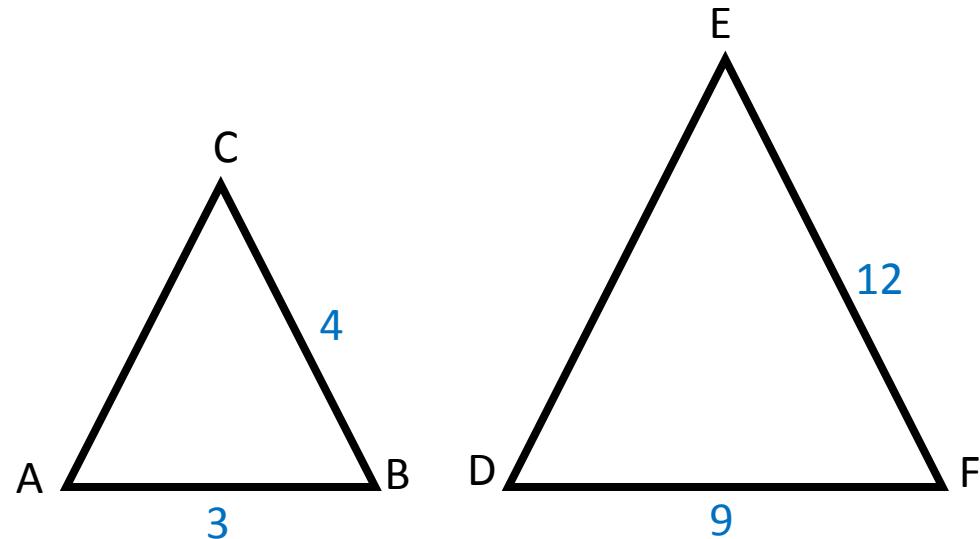
*2 is positive, so we have enlarged the shape!*

$$2 = \frac{x}{7}$$

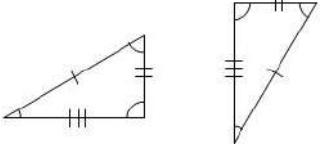
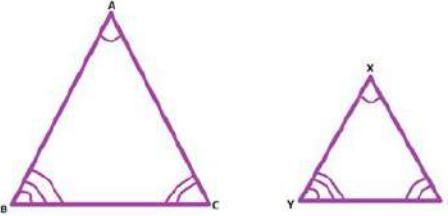
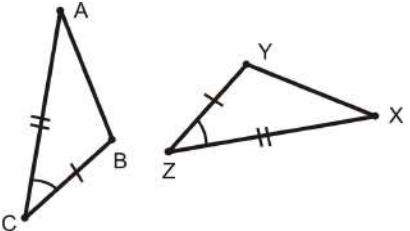
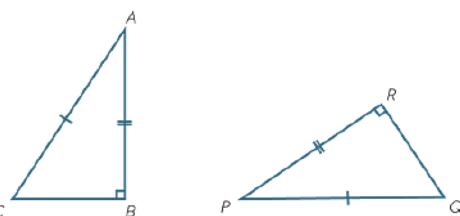
$$x = 2 \times 7$$

$$x = 14$$

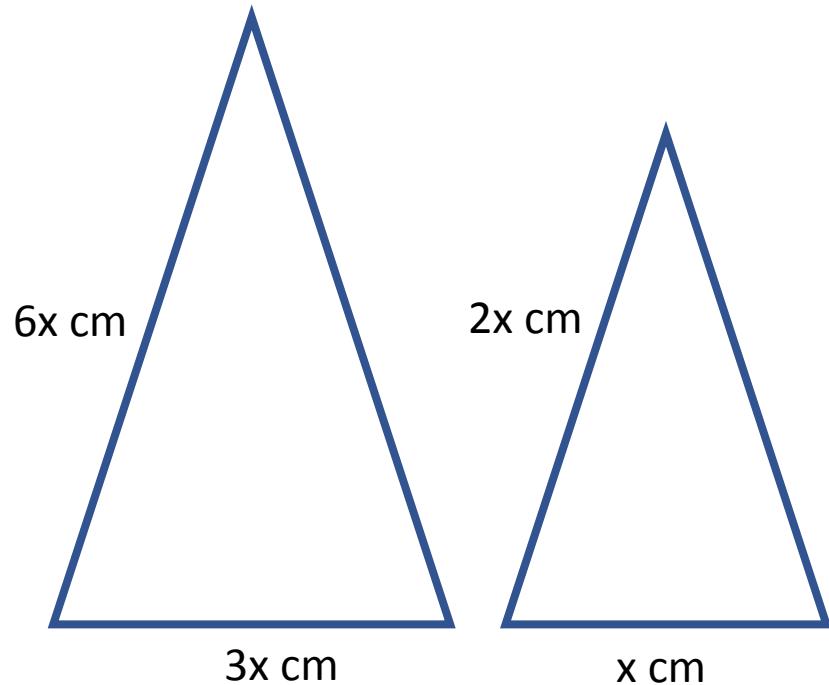
- Concepts of finding scale factors and unknown side lengths for similar triangles are the same for normal shapes.
- Two triangles will be similar if **corresponding angles are equal and corresponding sides are in the same ratio**.



To say that triangle ABC is similar to DEF, we write:  
 $\triangle ABC \parallel\parallel \triangle DEF$  or  $\triangle ABC \sim \triangle DEF$

Name	Definition	Diagram
<b>SSS (side, side, side)</b>	All three pairs of corresponding sides are in the same ratio.	
<b>AAA (Angle, Angle, Angle)</b>	All three corresponding angles are equal. (If two angles are the same, it can be assumed the third ones are the same as well)	
<b>SAS (Side, Angle Side)</b>	Two pairs of corresponding sides are in the same ratio and the included angle is equal. ( $24/12=2$ and $28/14 = 2$ )	
<b>RHS (Right Angle, Hypotenuse, Side)</b>	The hypotenuse of right-angled triangles and another corresponding pair of sides are in the same ratio.	

- Prove that the following triangles are similar



### 1. Identify the information given

- 2 pairs of corresponding sides
- 1 pair of corresponding angles
- SAS test can be used

### 2. Check if corresponding sides are in the same ratio, and angles are the same.

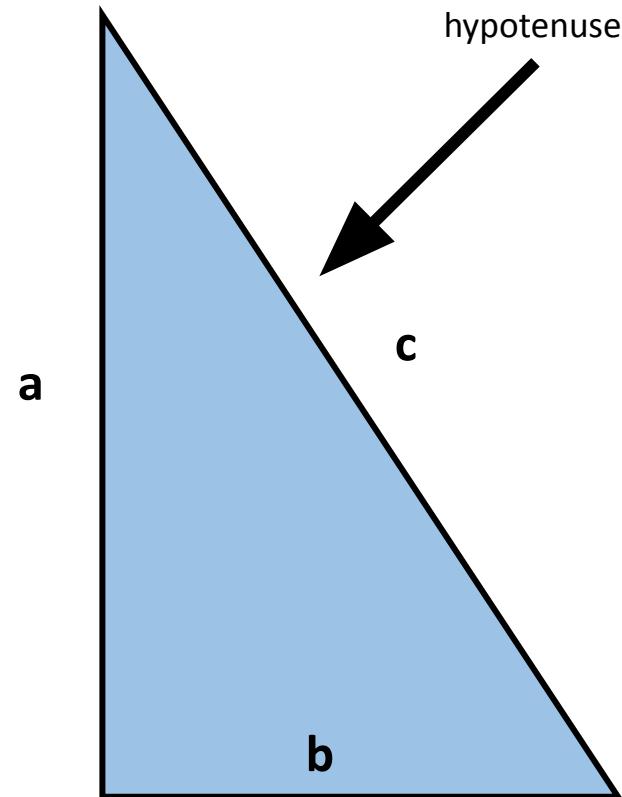
- Angles are the same
- Sides:  $6x/2x=3$ ,  $3x/x=3$ . Therefore, sides are in the same ratio.

### 3. Write a concluding statement by stating the test used.

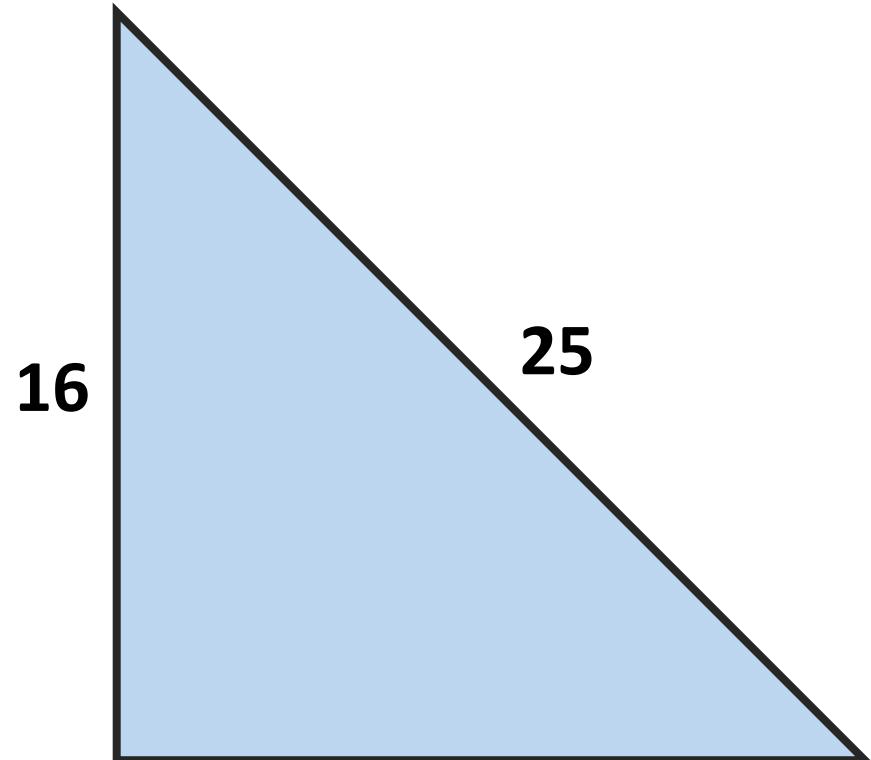
The corresponding angles are the same, and sides are in the same ratio. Therefore, by the SAS test, the triangles are similar.

- Allows us to find the length of any unknown side of a **right-angled triangle**.

$$c^2 = a^2 + b^2$$



Find the length of the unknown side of this triangle.



$$c^2 = a^2 + b^2$$

$$25^2 = 16^2 + b^2$$

$$b^2 = 25^2 - 16^2$$

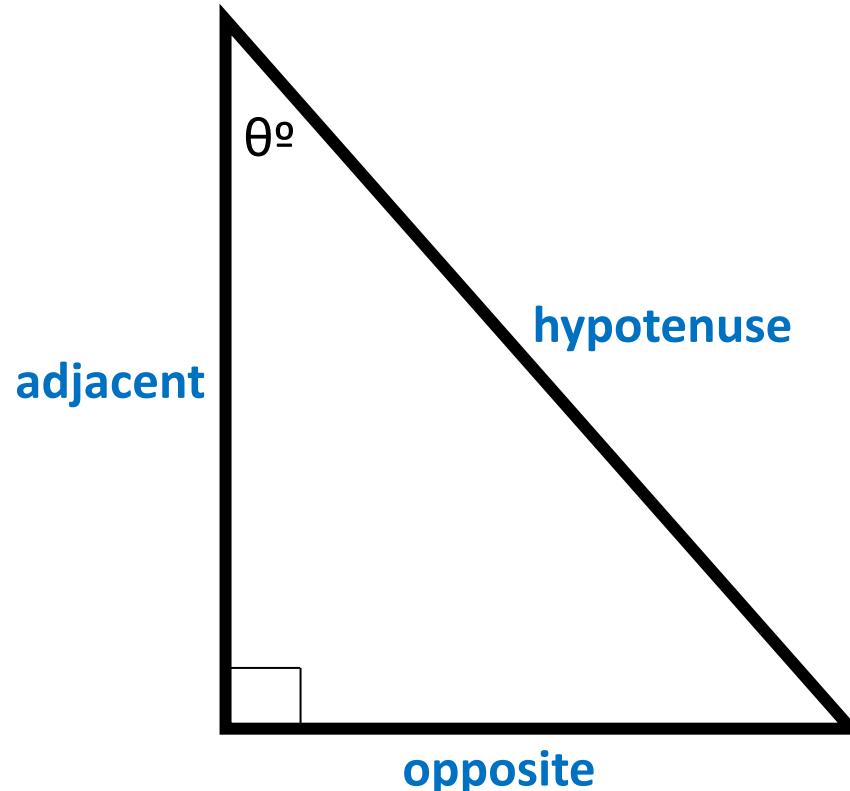
$$b^2 = 625 - 256$$

$$b^2 = 369$$

$$b = \sqrt{369}$$

$$b = 19.2$$

- Help us find side lengths/angles in a right-angled triangle
- The ratios are referenced according to an angle,  $\theta$  (theta).



*We can label  
each side of the  
triangle with  
reference to an  
angle.*

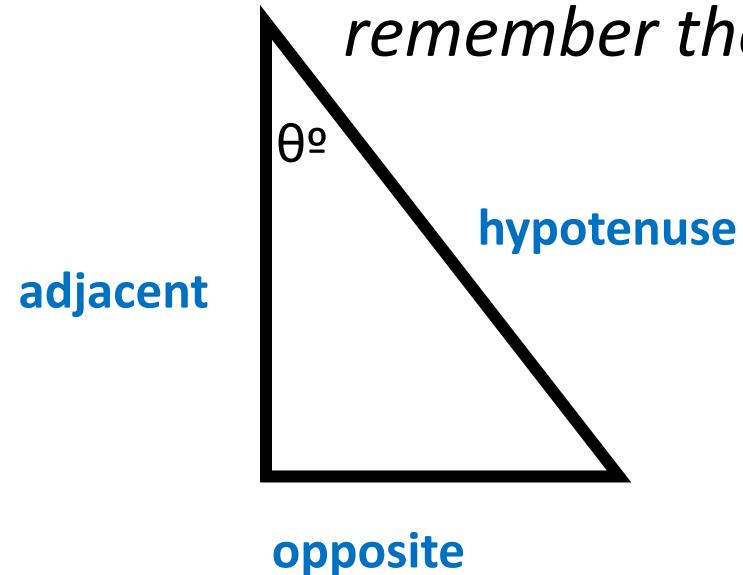
**Sine Ratio:**  $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$

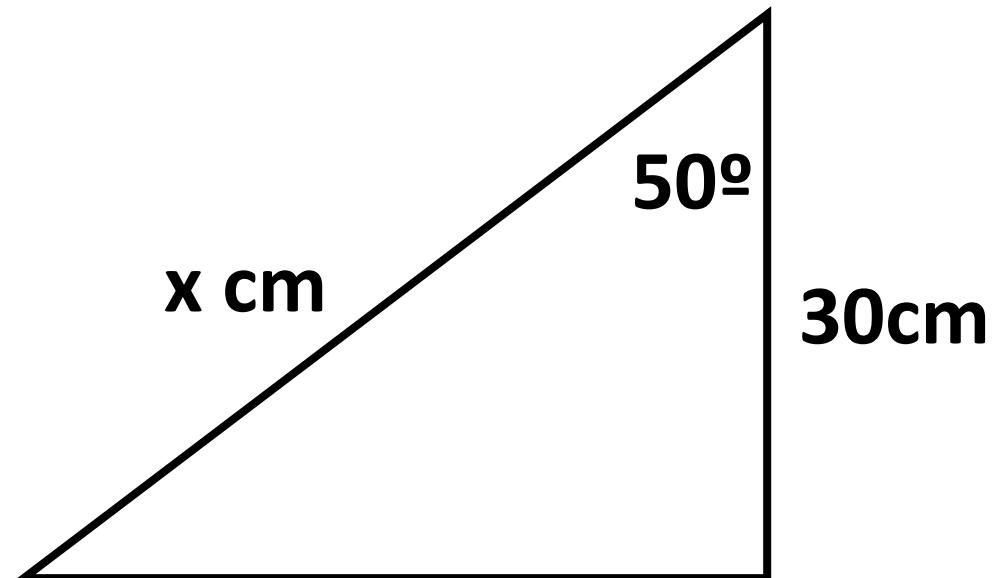
**Cosine Ratio:**  $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$

**Tangent Ratio:**  $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$

*How else can we represent the tan function?*

*Tip: Use the mnemonic **SOH CAH TOA** to help you remember these!*





Find the length of the unknown side:

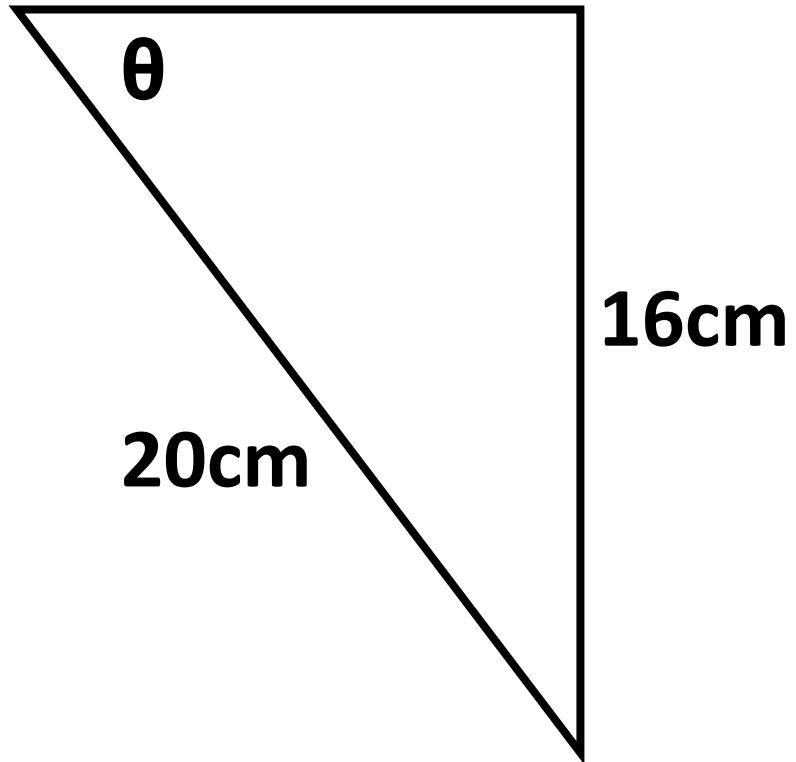
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos(50) = \frac{30}{x}$$

$$x = \frac{30}{\cos(50)}$$

$$x = 46.7 \text{ cm}$$

Make sure your calculator is in **degrees** mode!



Find the size of the unknown angle:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

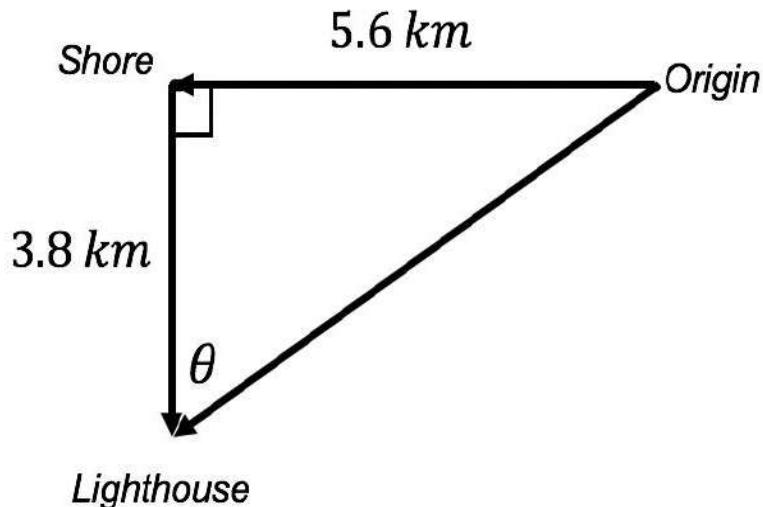
$$\sin(\theta) = \frac{16}{20}$$

$$\theta = \sin^{-1}\left(\frac{16}{20}\right)$$

$$\theta = 53.1^\circ$$

Make sure your calculator is in **degrees** mode!

- A ship starts at the origin and sails due west for 5.6 km to reach the shore and then due south for 3.8 km to reach a lighthouse. Find the angle between the shore and the origin, where the ship is now, and the shortest distance from the origin to the lighthouse.



**ANGLE:**

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan(\theta) = \frac{3.8}{5.6}$$
$$\theta = \tan^{-1}\left(\frac{3.8}{5.6}\right)$$
$$\theta = 34^\circ$$

**SHORTEST DISTANCE:**

$$c^2 = a^2 + b^2$$
$$c^2 = 5.6^2 + 3.8^2$$
$$c^2 = 31.36 + 14.44$$
$$c^2 = 45.8$$
$$c = 6.77 \text{ km}$$

Make sure your calculator is in **degrees** mode!

- Always draw a diagram when working with geometry questions, especially worded problems.
- Be very, very careful of units in geometry questions. Stay consistent with units when doing calculations.
- Learn how to convert between units
- Leave your answers in the units given in the question
- Be wary of decimal places
- Memorize your trigonometric ratios!

- The math involved with quantifying the chance of an event occurring.
- A probability is a number that lies between 0 and 1 (inclusive).
- Probabilities can be written as a percentage, fraction or decimal.

**Using the words *certainly, likely, equal, unlikely or impossible*, classify the chance of the following events occurring:**

- *Certain*
- *Equal*
- *Unlikely*
- *Impossible*

- **Sample Space** – The list of all possible outcomes of an event

*For example, the sample space of rolling a six-sided die is {1, 2, 3, 4, 5, 6}*

- **Event** – This is a subset of the sample space.

*For example, rolling an even number from a six-sided die*

- **Compound event** – two events occurring simultaneously

*This can be rolling an odd number greater than or equal to 3 from a six-sided die.*

$S=\{3, 5\}$

**Let's investigate these in an example!**

- **Probability of an event occurring**

$$Pr(\text{event}) = \frac{\text{number of outcomes favouring the event}}{\text{total number of outcomes}}$$

- **Complementary Events** – The chance of an event **not** occurring.

$$Pr(\text{not event}) = 1 - Pr(\text{event})$$

A letter was randomly selected from the word **MAGICIAN**.

1. List the sample space

$$S = \{M, A, G, I, C, I, A, N\}$$

2. What is the probability of selecting the letter A?

*There are 2 letter As in the word, and eight letters in total.*

$$\Pr(A) = \frac{2}{8} = \frac{1}{4}$$

3. What is the probability of choosing a vowel?

$$\Pr(\text{vowel}) = \frac{4}{8} = \frac{1}{2}$$

3. What is the probability of not choosing a vowel?

*This is a complementary event! The complement of not choosing a vowel is choosing a consonant.*

$$\Pr(\text{not choosing a vowel}) = 1 - \Pr(\text{choosing a vowel})$$

$$\Pr(\text{not choosing a vowel}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Don't forget to leave your fractions in the simplest form!

$\in$  means 'an element of.'

If you have a Set A, and 5 is an element in the set, you can write:  $5 \in A$ .

$\subseteq$  is the symbol for a **subset**.

All the elements of one variable that are contained in the sample space. E.g.  $\{1,3,5\} \subseteq \{1,2,3,4,5,6\}$

$\emptyset$  means 'empty set.'

This contains no elements.

$A'$  denotes the **complement** of event A

$n(A)$  refers to the **number of elements** in set A

$Pr(A)$  means the probability of A occurring.

Union ( $\cup$ ) means the elements in **multiple sets combined**.  
'and'

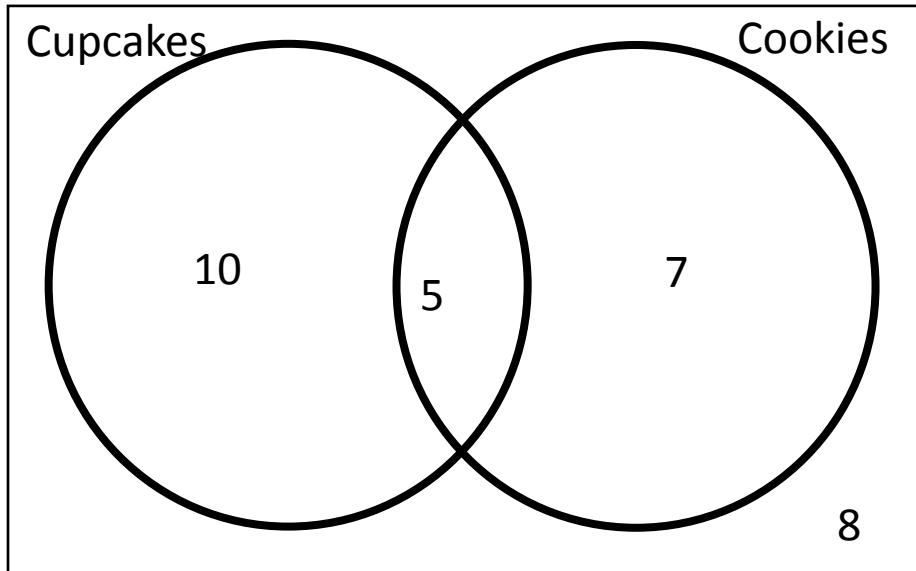
Intersection ( $\cap$ ) means the elements in multiple sets that are **common with each other**.

If  $A=\{1,3,4\}$  and  $B=\{2,3,6\}$

$$A \cup B = \{1,2,3,6\}$$

$$A \cap B = \{3\}$$

- A way of representing data that shows the relationship between two sets.



**Find the probability of randomly selecting a person:**

**Total people surveyed: 30**

*Who likes cupcakes only.*

$$\frac{10}{30} = \frac{1}{3}$$

*Who likes both cupcakes and cookies only.*

$$\frac{5}{30} = \frac{1}{6}$$

*Who likes cookies.*

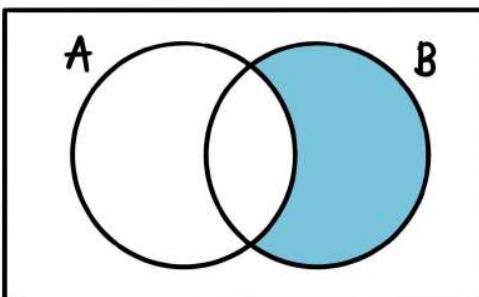
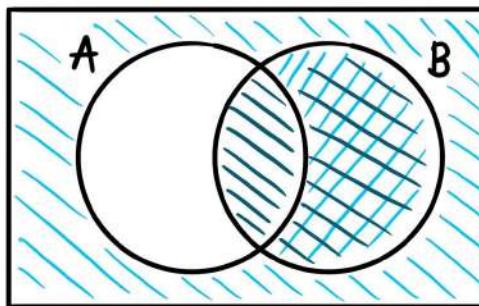
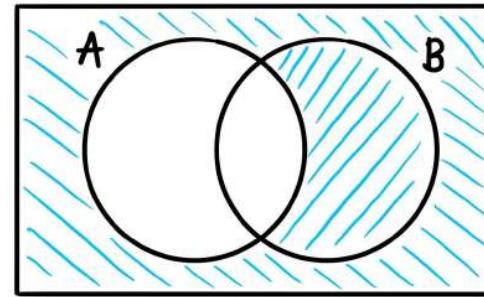
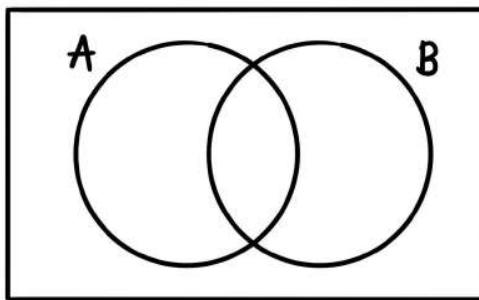
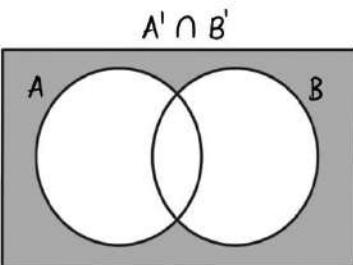
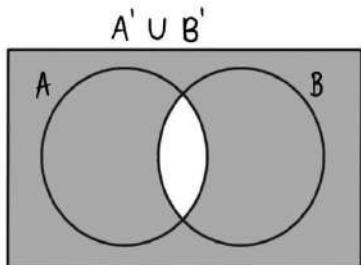
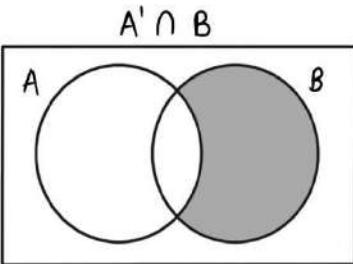
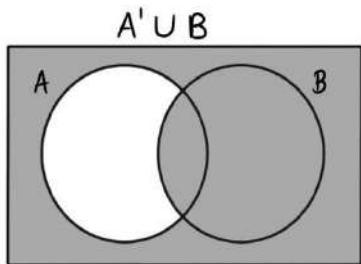
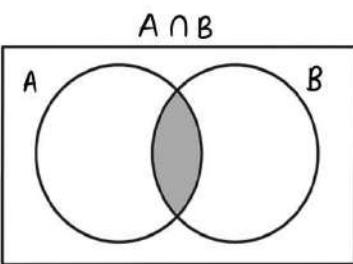
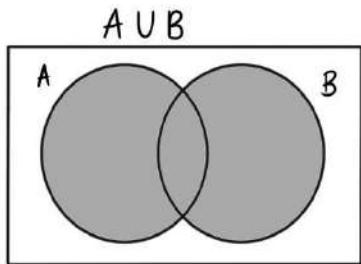
$$\frac{5+7}{30} = \frac{12}{30} = \frac{6}{15}$$

*Who likes neither cookies or cupcakes.*

$$\frac{8}{30} = \frac{4}{15}$$

# Chance and Data

## Set notation and venn diagrams



Let's derive  $A' \cap B$

- Another way of representing data
- Easily helps us see how people fall into different categories
- All rows and columns must add up to the value in the ‘totals’ section
- Can use this idea to fill in the blanks on empty tables
- Can also have a two-way table with probabilities, total would be 1

	Dancing	Don't like Dancing	TOTALS
Singing	32 This is the number of people who like both singing and dancing.	40 This is the number of people who only like singing.	72 This is the total number of people who like singing.
Don't like Singing	14 This is the number of people who only like dancing.	14 This is the amount of people who like neither dancing or singing.	28 This is the total number of people who don't like singing.
TOTALS	46 This is the total number of people who like dancing.	54 This is the total number of people who don't like dancing.	100 This is the total number of people that were surveyed.

- Two-way tables can also contain probabilities, instead of the cardinal number. The probabilities must always add up to one.

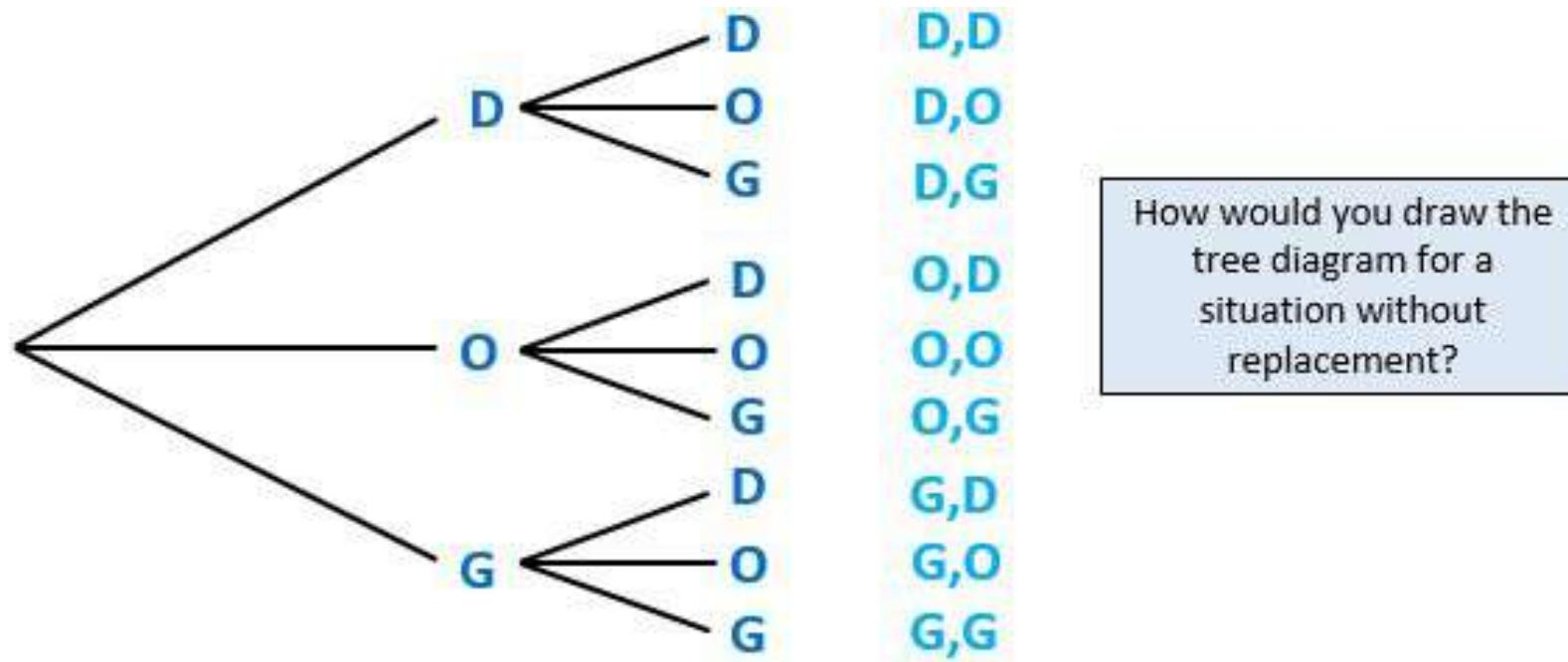
	A	A'	TOTALS
B	0.2	0.4	0.6
B'	0.1	0.3	0.4
TOTALS	0.3	0.7	1

Additionally, here's a set notation representation in a Two-Way Table.

	A	A'	TOTALS
B	$n(A \cap B)$ or $Pr(A \cap B)$	$n(A' \cap B)$ or $Pr(A' \cap B)$	$n(B)$ or $Pr(B)$
B'	$n(A \cap B')$ or $Pr(A \cap B')$	$n(A' \cap B')$ or $Pr(A' \cap B')$	$n(B')$ or $Pr(B')$
TOTALS	$n(A)$ or $Pr(A)$	$n(A')$ or $Pr(A')$	$n(\Omega)$ or 1

- Allows us to list a sample space for any scenario.

***Two letters was randomly selected from the word DOG with replacement. List the sample space.***



- Is determined from results of an experiment or survey
- This probability can be generalized to a similar situation

$$\text{Experimental Probability} = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in experiment}}$$

In sample of 100 people, 30 of them said they owned a cat.

What is the probability of randomly selecting a person that owns a cat?

$$\Pr(\text{cat}) = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in experiment}}$$

$$\Pr(\text{cat}) = \frac{30}{100} = \frac{3}{10}$$

In a sample of 500 people, how many people would be expected to own a cat?

$$\Pr(\text{cat}) = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in experiment}}$$

$$\frac{3}{10} = \frac{\text{number of times the outcome occurs}}{500}$$

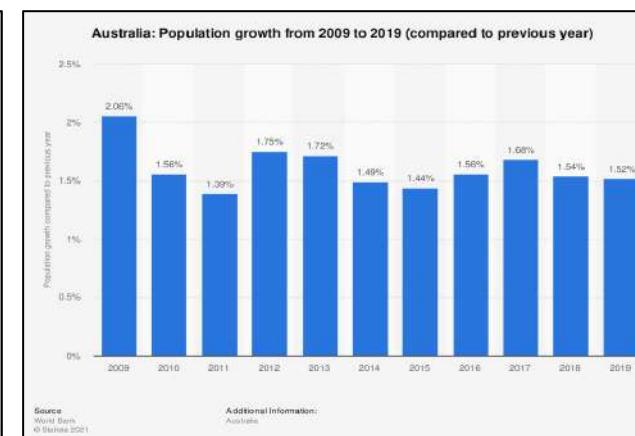
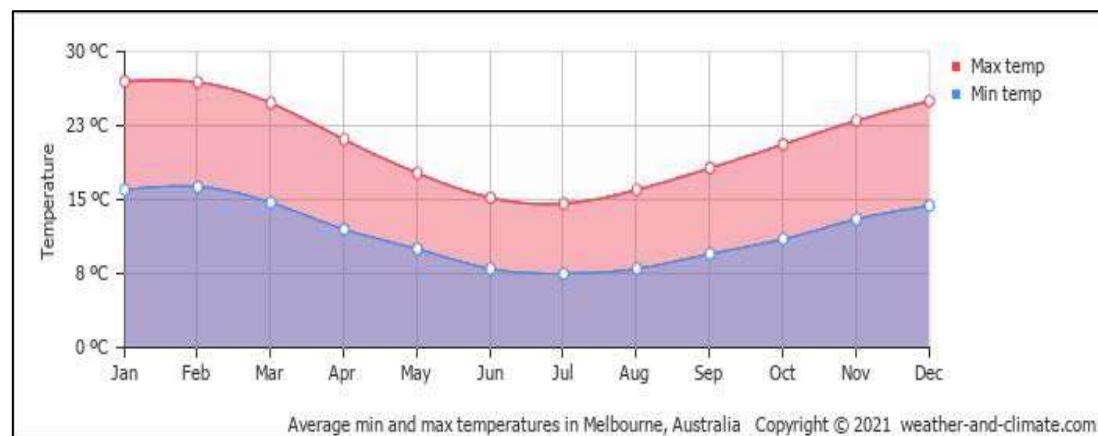
$$500 \times \frac{3}{10} = 150 \text{ people}$$

## What is data?

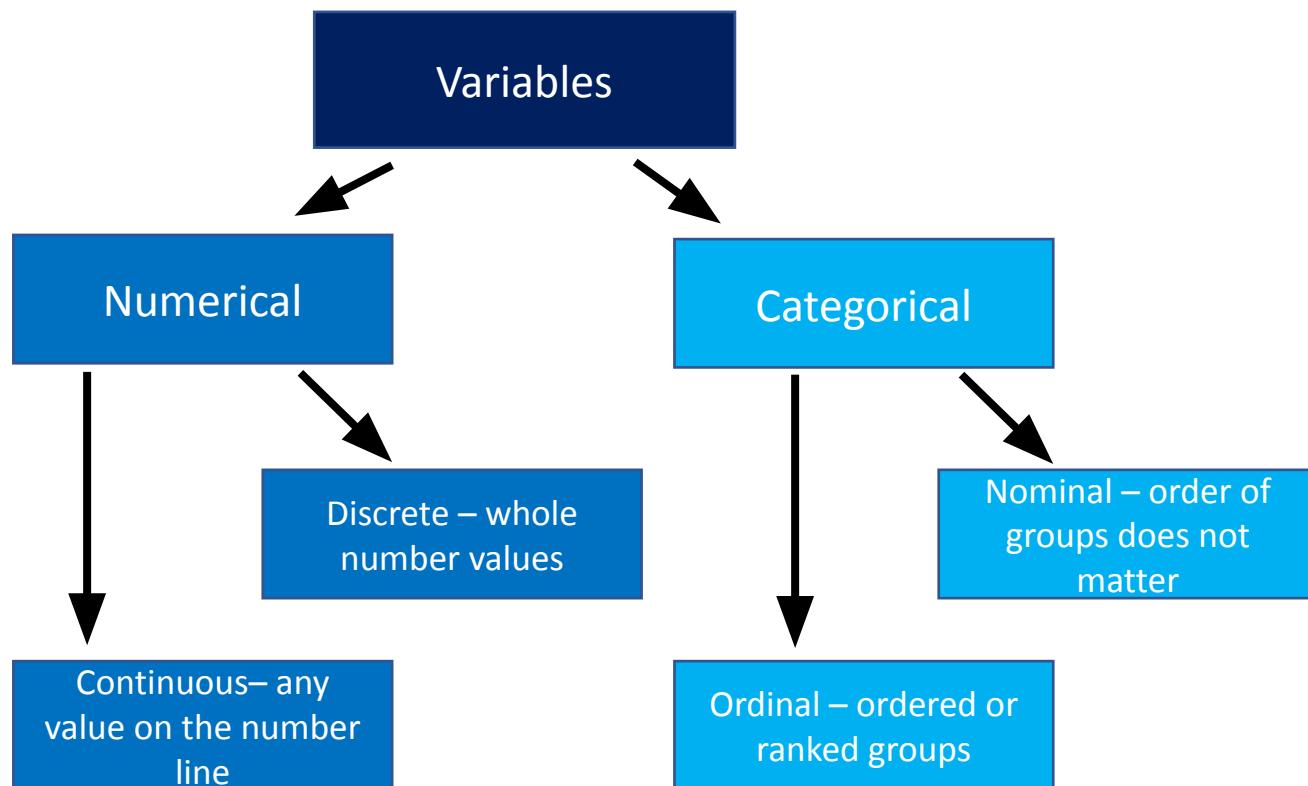
- Data can be defined as 'facts or statistics collected for analysis or reference.'
- It can be portrayed in different ways, such as graphs and stem and leaf plots.

## Here are some data that are commonly represented in different ways.

- Weather forecasts
- COVID case/vaccination data
- Population growth



- We will be looking at Univariate Data, which means **data of a single variable** (looking at how a single variable is changing)



### Discrete:

Number of people in your family

### Continuous:

Your height, temperature

### Nominal:

Favorite colors, favorite food, etc

### Ordinal:

Movie rating: terrible, okay, good, amazing

- We need to be able to look at a set of data given and analyze it. We usually do this by finding the following:

**Mean (Average):** We sum all the values and divide this by the number of values.

$$\text{Mean } (\bar{x}) = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\text{sum of all data}}{\text{number of data points}}$$

**Median:** The is the middle number in a set of values. If there are two middle values, take the average of them.

**Mode:** This is the value that appears the most in a data set.

**Range:** Gives an indication of the spread of a set of data. Affected a lot by outliers!

$$\text{range} = \text{highest data point} - \text{lowest data point}$$

Let's calculate the mean, median, mode and range for the following set of data.

**3 5 13 6 20 3 3 7 21**

**Mean:**

$$\frac{\text{sum of all data}}{\text{number of data points}} = \frac{3 + 5 + 13 + 6 + 20 + 3 + 3 + 7 + 21}{9} = \frac{81}{9} = 9$$

**Median:**

We need to find the middle number of this data set.

The numbers need to be put in **ascending order first!**

**3 3 3 5 6 7 13 20 21**

There are nine values, so the middle number will occur at the 5<sup>th</sup> position!

**3 3 3 5 6 7 13 20 21**

Therefore, the median is 6.

Let's calculate the mean, median, mode and range for the following set of data.

3 5 13 6 20 3 3 7 21

**Mode:** *The mode is the value that appears the most in this data set. In this case, the mode is 3.*

**Range:** *Lets look at our numbers in ascending order:*

3 3 3 5 6 7 13 20 21

$$\text{range} = 21 - 3$$

$$\text{range} = 18$$

*This indicates a reasonably large spread.*

- A student scores a 75, 79 and 83 on her first three math tests.
- **Calculate the average of her first 3 tests scores**

$$\frac{\text{sum of all data}}{\text{number of data points}} = \frac{75 + 79 + 83}{3} = 79$$

- **What does she need to score on her next math test to raise her average to an 82?**

$$\frac{\text{sum of all data}}{\text{number of data points}} = 82$$

$$\frac{75 + 79 + 83 + x}{4} = 82$$

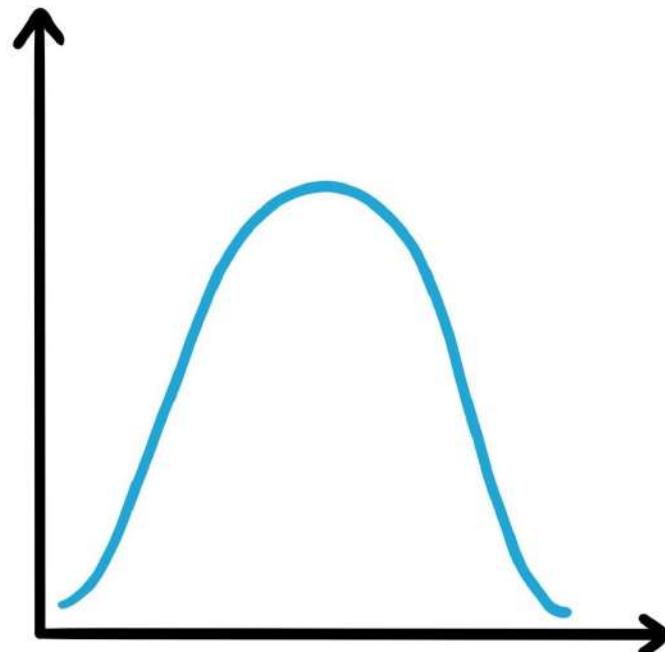
$$237 + x = 82 \times 4$$

$$237 + x = 328$$

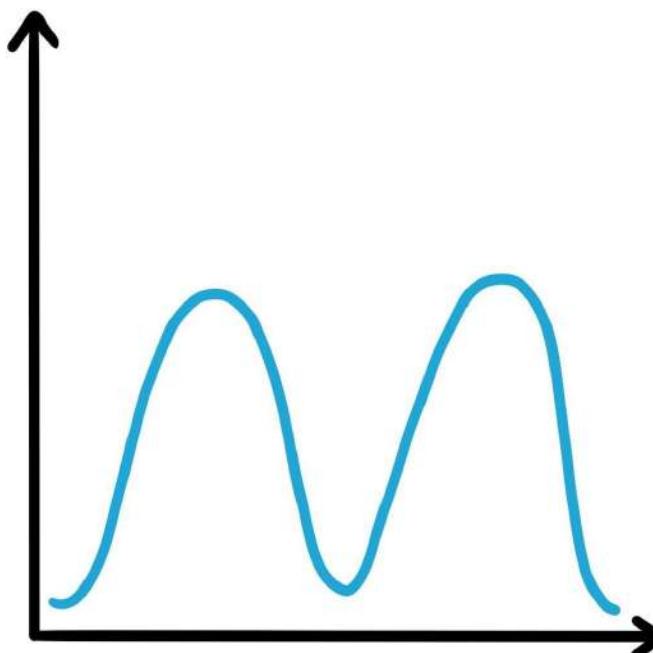
$$x = 328 - 237$$

$$x = 91$$

**Symmetric Distribution** – data is bunched around the center.



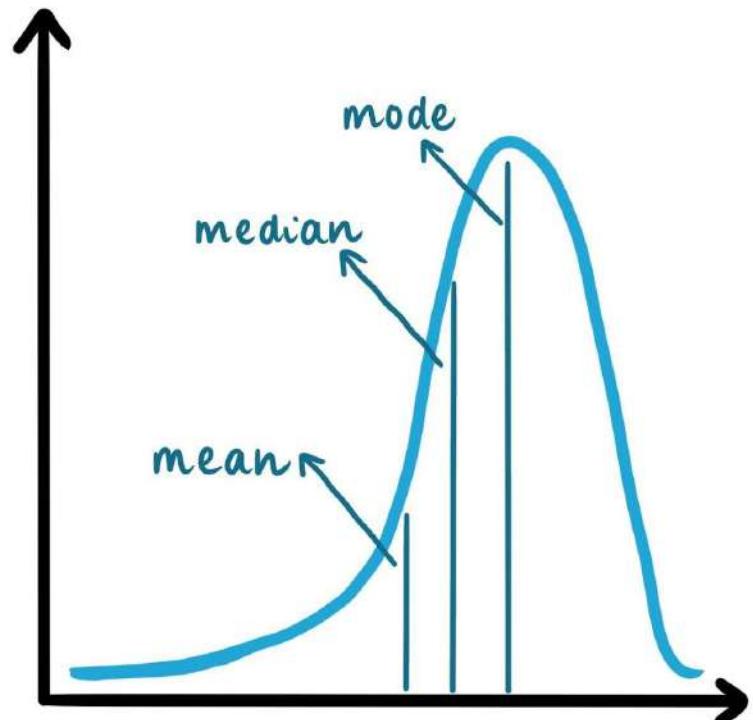
**Bi-Modal Distribution** – data has two distinct modes, or two maximum values.



The peak of the graph always represents the mode!

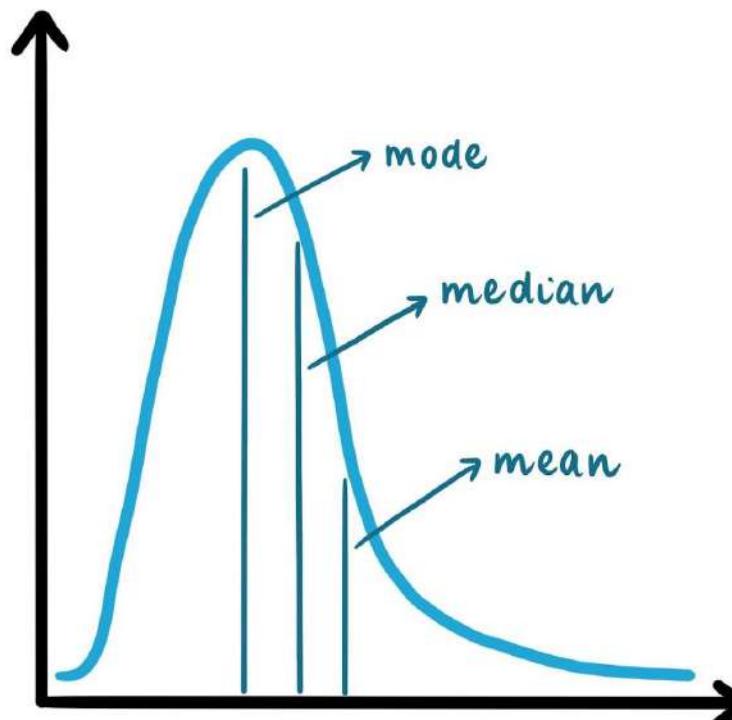
**Negative/Left Skew** – Tail of the graph is where the lower values are. Most of the data is to the right.

$$\text{mean} < \text{median} < \text{mode}$$



**Positive/Right Skew** – Tail of the graph is where the higher values are. Most of the data is to the right.

$$\text{mean} > \text{median} > \text{mode}$$

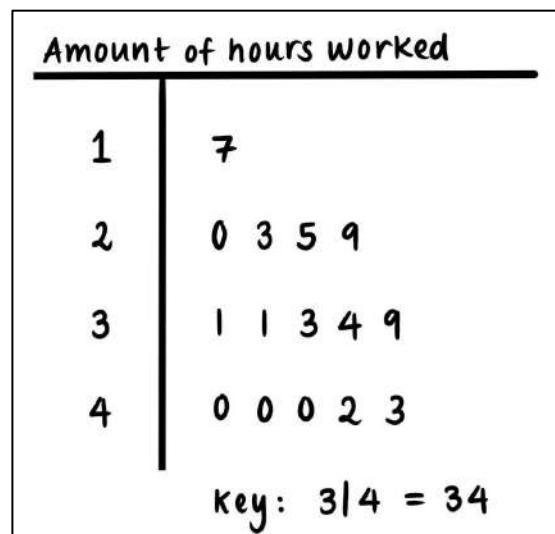
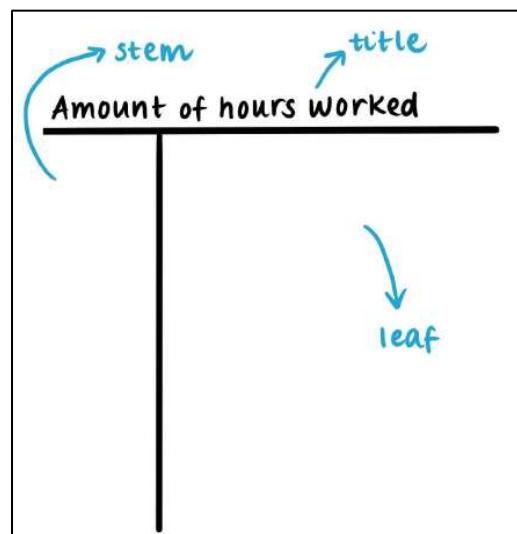


- A graphical display of a single set of data.

Let's construct a stem and leaf plot for the following scenario.

**15 people were asked how many hours they work a week on average.**  
*(In numerical order)*

17 20 23 25 29 31 31 33 34 39 40 40 40 42 43



- Let's figure out the type of distribution of the stem and leaf plot.

## Method 1: Finding the mean, median and mode.

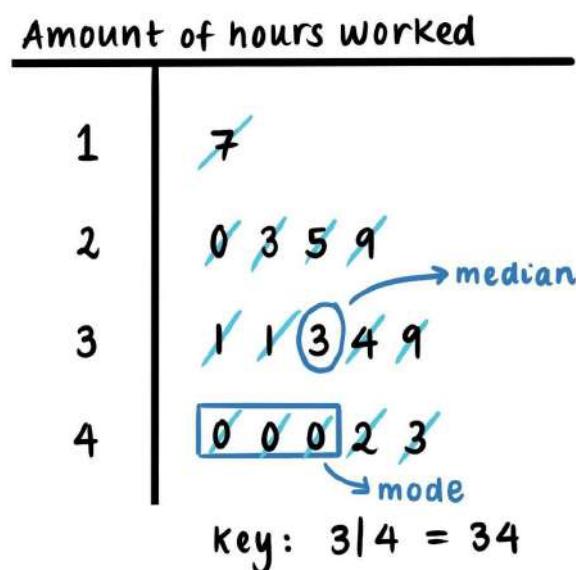
Median: 33

Mode: 40

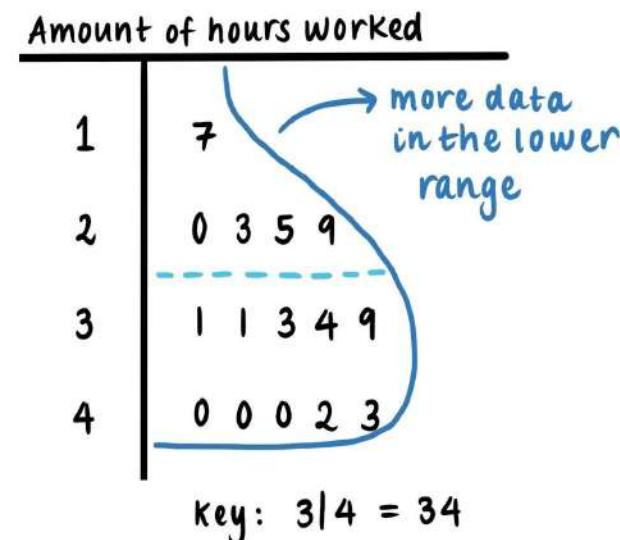
Mean:

$$\frac{17+20+23+25+29+31+31+33+34+39+40+40+40+42+43}{15} = 32.47$$

Since **mean < median < mode**, we have a negatively skewed distribution.



## Method 2: By inspection



- Is a way to represent relative frequencies of **discrete data**.
- Horizontal axis represents groups
- Vertical axis contains the frequency

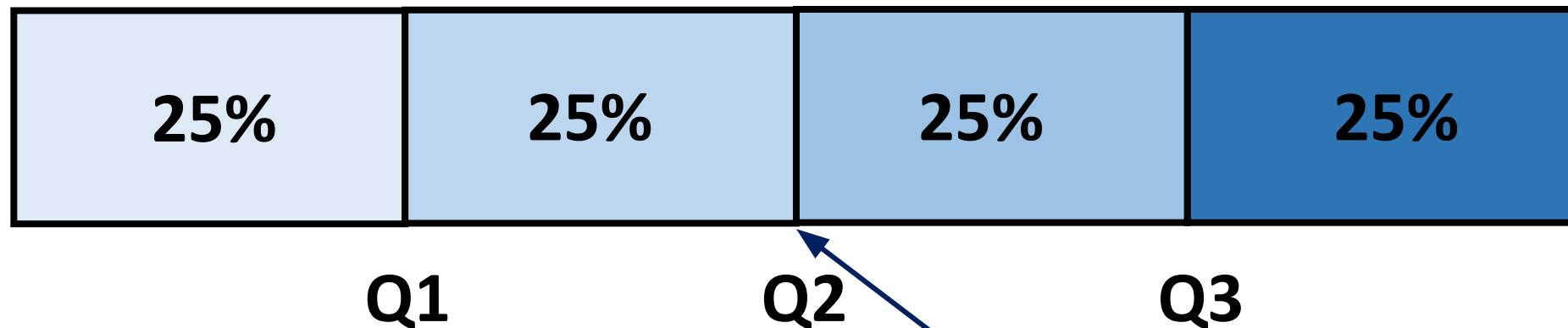
The number of donuts sold in a café was recorded for 10 days. Record the data in a histogram.

21 34 36 22 56 54 52 55 20 16

Range	Frequency
0-9	0
10-19	1
20-29	3
30-39	2
40-49	0
50-59	4

- The interquartile range is the range where the **middle 50%** of the data in a set lies.
- We need to arrange the data in ascending order and divide it into four equal parts

Interquartile Range



**Interquartile Range = Q3 - Q1**

Let's find the interquartile range of the following set of numbers:

1 1 3 5 6 8 8 10 10

1 1 3 5 6 8 8 10 10

**Quartile 1 (median of first half)**

$$\frac{3+1}{2} = 2$$

**Quartile 3 (median of second half)**

$$\frac{10+8}{2} = 9$$

**Interquartile Range**

$$Q3 - Q1 = 9 - 2 = 7$$

- Underline ALL the keywords in a probability question. Take special note if the question says and/or.
- Make sure you always leave probabilities in simplest form.
- Practice converting between fractions, decimals and percentages
- Become confident in using the notation for probability
- If you're stuck, try replicating the probability scenario in real life
- Practice analyzing graphs you may see in the media

- Make sure you understand the concepts before doing practice questions
- Complete practice questions regularly
- Create summary sheets to help you remember things
- Practice a variety of different question types
- Don't be afraid to ask for help – from peers, teachers etc
- Make sure you know how to use your calculator but do not rely on it!
- Don't forget to look after yourself

- Check over your work as you go or at the end
- Don't forget to keep track of time
- Show all your working out
- Underline key words, especially in lengthy worded questions
- Keep track of units, decimal places and signs
- Make your final answer stand out

## Key skills

### **Part 1 – Number and Algebra**

- Index Laws
- Algebraic Expressions
- Linear Graphs/Equations

### **Part 2 – Geometry**

- Area, Surface Area and Volume
- Similar Figures
- Pythagoras and Trigonometry

### **Part 3 – Chance and Data**

- Probability Introduction
- Probability Tools/Diagrams
- Data Representation

### **Part 4 – Tips**

- Study Tips
- Exam Tips

## Reminders

- Thank you for coming!

Questions?